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SENSITIVITY ANALYSIS OF A BOND MUTUAL FUND TO YIELD CURVE

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Warning

The "International Institut International des Assurances" and the "Institut de Science Financière et d'Assurance" do not intend to give any approval or disapproval to the opinions expressed in this master dissertation; they must be considered as specific to the author.

Dedication

To my wife, my children, my family, and my friends....

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Abbreviations and Acronyms

Acronym	Designation
AMC	Asset Manager Company
BDEAC	Development Bank of Central African States
BEAC	Central Bank of Central African States
BVMAC	Central African Stock Exchange
CAA	Autonomous Sinking Fund
CAR	Central African Republic
CD	Certificate of Deposit
CEMAC	Economic and Monetary Community of Central Africa
COBAC	Central African Banking Commission
COSUMAF	Financial Market Supervisory Commission of Central Africa
CRCT	Securities Settlement and Custody Unit
CRDV	Regional Securities Depository
ECF	Extended Credit Facility
EG	Equatorial Guinea
ES	Expected Shortfall
ESG	Economic Scenario Generator
FAGACE	African Guarantee and Economic Cooperation Fund
GAAP	Generally Accepted Accounting Principles
IFC	International Finance Corporation
IIR	Internal Rate of Return
ISIN	International Securities Identification Number
NAV	Net Asset Value
OTC	Over The Counter
OTZ	Zero-Coupon Treasury Bond
P&L	Profit and Loss
PMP	Weighted Average Price
Repo	Repurchase
SRC	Debt Collection Agency
SVT	Primary Dealer
TIAO	Interest Rate on Tenders
TMP	Weighted Average Interest Rate
UCITS	Undertaking for Collective Investments in Transferable Securities
VaR	Value at Risk
WAEMU	West African Economic and Monetary Union
YTM	Yield To Maturity

Content

Résumé

La construction d'une courbe de taux sans risque est au cœur de la valorisation des portefeuilles de titre à revenu fixe. Il s'agit d'une fonction qui associe à chaque maturité un taux de rendement d'un titre sans risque. Dans la zone CEMAC, la valorisation des fonds communs de placement (monétaires, obligataires, diversifiés ou actions) n'est pas encore encadrée par un règlement de la COSUMAF qui est l'autorité de marché. En absence d'un dispositif réglementaire, les praticiens considèrent que la valeur d'un actif à revenu fixe correspond au prorata du flux de trésorerie à obtenir à l'échéance. Il s'agit de considérer que la valeur de ce titre est la somme du nominal et du coupon couru non échu. Cette approche considère ainsi que la courbe des taux a un déplacement plat à travers le temps. En d'autres termes, le taux de rendement servi est égal au taux de coupon d'une période à une autre.

Toutefois, les décotes importantes enregistrées dans le cadre des adjudications organisées par la BEAC et celles observées sur les échanges des titres publics du Congo, du Gabon, du Tchad ou de la Guinée Équatoriale montrent que le rendement exigé par les investisseurs ne correspond pas au taux de coupon. Ce qui peut induire une surévaluation ou une sous-évaluation de ces titres. De plus, ces rendements changent significativement d'une période à une autre en fonction des besoins de financement des États membres, des perspectives économiques et financières des pays membres et de la perception des investisseurs sur le risque du pays émetteur. Compte tenu de tous ces facteurs, la valorisation des actifs et des fonds communs obligataires ou monétaires au prorata des flux de trésorerie futurs peut laisser apparaitre des différences de valorisation significatives par rapport à leur valeur actuelle réelle basée sur la courbe des taux.

L'objectif central de ce mémoire est d'analyser la sensibilité d'un fonds obligataire à la courbe des taux. Il s'agit précisément d'analyser les écarts de valorisation entre l'approche utilisée par le marché et une approche basée sur les courbes de taux. Ainsi, un objectif spécifique de ce travail est de construire la courbe des taux sans risque de chacun des pays de la CEMAC. Contrairement aux courbes de taux mensuel publiées par la BEAC, les courbes de taux que nous construisons dans ce mémoire sont hebdomadaires dans la mesure où le calcul de la valeur liquidative des fonds est hebdomadaire. Un autre objectif spécifique de cette étude est faire une analyse comparative des différences de valorisation des fonds qui apparaissent en utilisant les courbes de taux de la BEAC et celles qui apparaissent en utilisant nos courbes de taux calculées. Le dernier objectif de ce travail est d'examiner l'impact de ces différences d'estimation sur la valeur de l'actif net et de la performance du fonds.

Les courbes de taux sont calculées en utilisant le modèle de Nelson et Siegel (1987) et le modèle de Svensson (1994) à partir des données sur les émissions et les échanges de BTA et d'OTA. La partie courte de la courbe des taux est quant à elle ajustée en utilisant les pensions interbancaires adossées sur les titres publics de la CEMAC. Afin d'examiner l'étude de la sensibilité du fonds, nous calculons la valeur à risque des différences de valorisation en s'appuyant sur une approche paramétrique d'une part et une approche Monte Carlo d'autre part. Les données proviennent du fonds commun de placement CRBC PROSPERITE.

Abstract

Building a risk-free yield curve is at the heart of the valuation of fixed income portfolios. This corresponds to a function that associates the interest rate of a risk-free security to a tenor. In the CEMAC zone, the valuation of mutual funds (monetary, bond, diversified or equities) is not yet regulated by a regulation of the COSUMAF which is the market authority. Without a regulatory framework, practitioners consider that the value of a fixed income asset corresponds to the pro rata of the cash flow to be obtained at maturity. They consider that the value of this security is the sum of the nominal value and the accrued coupon not yet due. This approach thus considers that the yield curve has a flat displacement over time. In other words, the yield is equal to the coupon rate from one period to another.

However, the significant discounts recorded within T-Bonds and T-Bills auctions and those observed on the transactions of government securities from Congo, Gabon, Chad or Equatorial Guinea show that the required yield by investors does not correspond to the coupon rate. This can lead to an overpricing or underpricing of these securities Moreover, yields significantly change from one period to another depending on the financing needs of member states, their economic and financial outlook, and investors' perception of the risk of the issuing country. Considering these elements, the valuation of fixed income securities and mutual funds as proportion of future cash flows may conduct to significant valuation differences from their present value based on the yield curve.

The central objective of this master thesis is to make a sensitivity analysis of a bond mutual fund to the yield curve. More specifically, we analyze valuation differences between the approach used by practitioners and an approach based on yield curves. Thus, a specific objective of this work is to construct the risk-free rate curve for each CEMAC country. By contrast to monthly curves published by the BEAC, we extract weekly yield curves insofar as the calculation of the net asset value per share of mutual funds is weekly achieved. Another specific objective of this study is to make a comparative analysis of valuation differences which appear using the BEAC yield curves and those which appear using our calculated yield curves. The final objective of this work is to examine the impact of these valuation differences on the net asset value and the performance of the fund.

Yield curves are computed using the Nelson and Siegel's (1987) model and the extended version of Svensson (1994) from market data of T-Bonds and T-Bills issues. The short area of the yield curve is adjusted using interbank repos backed by CEMAC government securities. In order to examine the sensitivity study of the fund, we calculate the value at risk of valuation differences using a parametric approach on the one hand and a Monte Carlo approach on the other hand. Data comes from the CRBC PROSPERITE mutual fund.

Introduction

Yield curve or term structure of interest rates is at the center of fixed income markets. It plays a central role in pricing fixed-income securities. The term structure is helpful for market participants, informing them whether a security is temporarily overpriced or underpriced. More specifically, if a security's rate of return is above the yield curve, this indicates that the security is underpriced. Inversely, if the rate of return lies below the yield term structure, then it shows that the security is overpriced. The yield curve also serves for risk management and for national central banks that conduct monetary policy through short interest rates. For insurance companies, it is the most important tool to construct the best estimates for both assets and liabilities.

There has been an increased use of local bond markets in African countries, mainly after the 2008 global financial turmoil since external funding were rare. The amount of sovereign local currency bonds issued by government reached USD 200 billion per year since 2009 [see Mezui (2021)]. This increase of government securities issues has positively contributed to the developments of collective management activity which is experiencing a great expansion in African countries. The development of capital market is favorable to the development of this activity. Indeed, increasing initial public offerings (IPO) and recurring sovereign and private issues on the other hand are real drivers that stimulate the professionalized management of savings.

While other African countries are benefiting from growths in all capital markets, Central Africa's investment horizon is currently limited due to the sluggishness of its stock market. Only the regional market of public securities with open subscription offers significant opportunities to an asset management company (AMC). In the Central African Economic and Monetary Community (CEMAC), existing mutual funds are mainly invested in fixed income securities markets. Their valuation is not yet guided by a regulation of the Financial Market Supervisory Commission of Central Africa (COSUMAF) which is the market authority. Without a regulatory framework, practitioners consider that the value of a fixed income asset corresponds to a proportion of future cash flows at maturity. More specifically, the value of a fixed income security is the sum of the nominal value and the accrued coupon not yet due. Therefore, this approach suggests that the yield curve has a flat displacement over time, i.e., the yield is equal to the coupon rate from one period to another.

However, the significant discounts recorded within T-Bonds and T-Bills auctions and those observed on the secondary market for government securities of Congo, Gabon, Chad or Equatorial Guinea show that the required yield by investors does not correspond to the coupon rate. This can lead to an overpricing or underpricing of these securities Moreover, yields significantly change from one period to another depending on the financing needs of member states, their economic and financial outlook, and investors' perception of the risk of the issuing country. Considering these elements, the valuation of fixed income securities and mutual funds as proportion of future cash flows may conduct to significant valuation differences from their present value based on the yield curve.

The main research question of this dissertation is to examine whether and the extent to which the yield curve impacts bond mutual funds valuation. Therefore, the central objective of this master thesis is to make a sensitivity analysis of a bond mutual fund to the yield curve. More specifically, we analyze valuation differences between the approach used by practitioners and an approach based on yield curves. Thus, a specific objective of this work is to construct the risk-free curve for each CEMAC country. By contrast to monthly curves published by the Central Bank of Central African States (BEAC), we extract weekly yield curves insofar as the computation of the net asset value per share of mutual funds is weekly achieved. Another specific objective of this study is to make a comparative analysis of valuation differences which appear using the BEAC yield curves and those which appear using our calculated yield curves. The final objective of this work is to examine the impact of these valuation differences on the net asset value and the performance of the fund.

Different quantitative tools and methods are employed in this study. We extract yield curves using parsimonious Nelson and Siegel model and the extended version of Svensson. These empirical models are suitable for an economic interpretation of the yield curve. Second, we examine the effects of yield curves on bond mutual funds using various mathematical tools related to interest rates and risk measures.

The dissertation is organized as follows. Chapter I describes the investment environment in the CEMAC zone. We present the main fixed income instruments issued and traded in the monetary union of the CEMAC zone. Chapter II makes a theoretical background of models explaining different shapes of the yield curve. In this chapter, we also make a review of existing techniques and mathematical models for fitting and estimating the term structure. Chapter III fits and estimates yield curves of CEMAC countries and the last chapter examines the impact of the yield curve on a bond mutual fund.

Chapter I | Investment Environment in the CEMAC Zone

Fixed-income markets are populated with a vast range of securities. In the monetary union of the Central African Economic and Monetary Union (CEMAC), three fixed income markets coexist: the regional stock market, the regional government securities market via subscription, and the Cameroon's market of negotiable debt instruments.

The Financial Market Supervisory Commission of Central Africa (COSUMAF hereafter) is the market authority of the regional market while the regional central securities depository is the central bank. The market animation is achieved by the Central African Stock Exchange (BVMAC) initially located in Libreville. Douala is the headquarters of the BVMAC while the market regulator is located in Libreville. Although the regulatory framework is set, the market however remains embryonic with rare corporate issues.

The regional government securities market is organized by the central bank is organized by the central bank, for the issuance of T-Bonds and T-Bills via open subscription. This market aims to freeze the direct financing to governments by the central bank. Regulation n°03/08/CEMAC/UMAC/CM enacted in October 06, 2008 set the regulatory framework for fungible treasury bonds through bid invitation to licensed primary dealers (SVT hereafter). The central bank organizes auctions through its national offices and each SVT has a depository account with the Securities Settlement and Custody Unit (CRCT hereafter) that also ensures securities' management. The different national treasuries operate as issuing agents to achieve their respective countries' budget deficits and debt management requirements.

The regulatory framework of the market of negotiable public instruments is set by Decree °94/611/PM enacted in December 30, 1994. Ministry of Finances through the Autonomous Sinking Fund is the market authority. It defend the issuance and amortization policy of treasury values.

Regulation n°44/CEMAC/UMAC/CM taken in March 27, 2015 establishes the legal framework for issuances and management of negotiable debt securities in the CEMAC zone. They are traded on organized markets (the regional government securities or the regional stock market) or over the counter (OTC).

In all fixed income markets, secondary trading are weak due to passive strategy of domestic banks which buy and hold debt instruments because of limited lending opportunities, limited supply of securities and prudential requirements.

This chapter presents the main fixed income instruments issued and traded in the monetary union of the CEMAC zone. In the next section, we present bonds markets while the second section describes money markets.

1. Bonds Markets

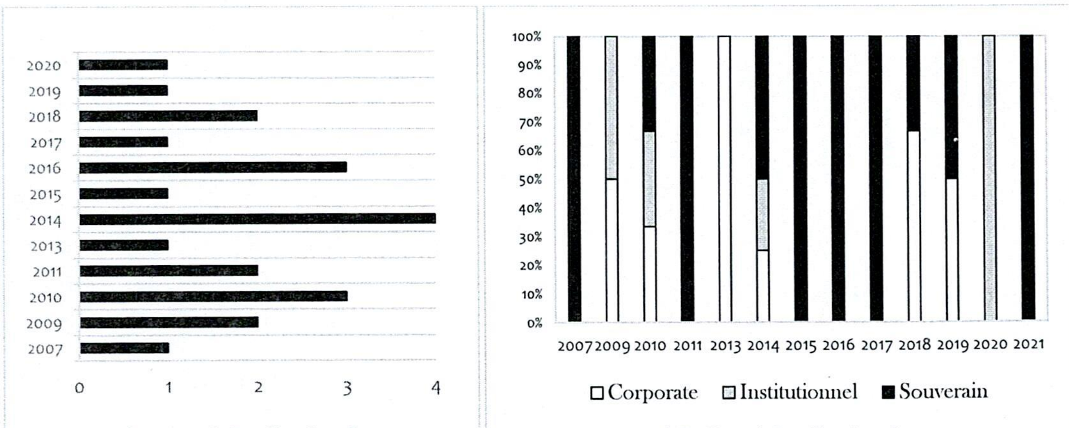
In the CEMAC zone, there are three types of bonds issues: bond syndication, bond auction, and zero-coupon bonds. Bond syndication mainly refers to issuances on the regional stock market while bond auctions are negotiable debt securities issued on the regional government securities markets. Government bonds auction are fungible, they can be reattached to an initial issue.

1.1. Bond Syndication

On the regional stock market, bond issues can be fulfilled through private placement or public offering. Only qualified investors such as pension funds, banks, brokerage firms and national social security funds are involved within a private placement. Many institutional and corporate. Several private placements have been recorded in the CEMAC zone compared to public offering placement[1].

By contrast, all investors (foreign and domestic) have free access for bond issues through syndication with public offering. However, the regional bond market remains underdeveloped. The number of per year bond issues is very weak and does not exceed 5 (see Figure 1, Panel a).

Figure 1 | Bond Syndication Issues



Panel a. Bond Syndication Issues

Panel B. Bond Syndication Issuers

Source: CRCT/Author's Calculations

Many factors could explain the sluggishness of bond syndication. First, the weakness of the banking system and the undercapitalization of several banks. Second, a rudimentary payment system in the CEMAC zone is a serious drawback to the development and

[1] For example: the BDEAC private placement in 2007, the City of Douala inaugural medium term notes programme in 2005, and the Afriland first bank bond issue in 2016.

integration of the financial sector and the economies of member countries. The payments system is slow, unreliable, complicated and costly thereby constituting a constraint on regional financial integration. Third, the credibility of issuers is lacking given that the member States do not have monetary sovereignty thereby leaving the Central Bank as the only risk-free issuer. Fourth, the unreliable and generally poorly structured information on markets results undermines market transparency which is in turn reflected in the low local demand for securities.

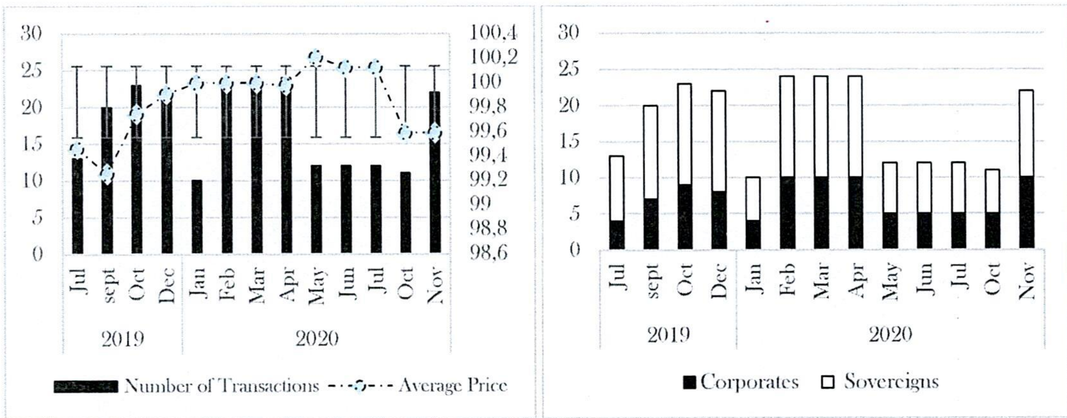
Figure 1, Panel b also shows that issuances on the primary market are highly dominated by sovereigns. Funds raised by member states were mainly destined to support developing projects including infrastructure, energy, education and health. The 2021 bond issue enabled Congo to repay partially the existing debt. Issues from international or regional institutional institutions such as: the African Guarantee and Economic Cooperation Fund (FAGACE), the International Finance Corporation (IFC), and the Development Bank of Central African States (BDEAC) intervene to sustain development projects (buildings, infrastructures, energy, etc.) and support private investments.

Corporate issues are marginal. They are impeded by lack financial information, excessive issuance costs, and asymmetric taxation. The lack of credible financial statements is the main reason why corporate issues are not widespread in the monetary union of central Africa. They cannot provide a realistic valuation of their financial health and their loan repayment capacity. The transactions costs can also reach at 4.35 percent of the amount issued on the primary market. More specifically, bond syndication comprises arrangement fees (2 percent), placement fees (1.25 percent), underwriting fees (0.75 percent), lead arranger fees (0.30 percent) and agent fees (0.05 percent). Moreover, there is asymmetry in the taxation system for bond issuances in the CEMAC zone. Obligations issued by the private sector are taxed while those issued by public sector are not. More specifically, Regulation n° 14/07-UEAC-175-CM-15 exempts from income tax on securities or any other tax interests on government bonds for CEMAC residents and interests on CEMAC local authorities' bonds. Funds raised by corporate are mainly destined to acquire industrial goods, to enforce solvency ratios for financial firms and extend the existing capacities.

Syndicated bonds in the CEMAC zone are traded both on the regional stock exchange and OTC but all secondary market transactions must be conducted through licensed brokerage firms. They also operate as clearer, trader, account keeper, custodian, and assets manager. Officially, it is the twelfth brokerage firm authorized to operate on the regional stock market. Nine have their headquarters in Cameroon (Afriland Bourse & Investissements S.A, Upline Securities Central Africa S.A (USCA), Attijari Securities Central Africa (ASCA), Financia capital, EDC Investment Corporation, Société sahélienne d'intermédiation financière de l'Afrique centrale (SAIFAC), Global Trade International Investment (GTI), Société générale – capital Securities central Africa (SG capital CEMAC) and Africa Bright Securities. The remaining three are CBT Bourse (Chad), LCB Capital (Congo), and BGFI Bourse (Gabon).

We can see that the regional stock market is not yet efficient. Prices are not correlated with the number of transactions. We can also see that there is no price volatility. The standard deviation remains constant from one month to another. Figure 2 also reveals that sovereigns bonds are more traded than corporates bonds including institutions and firms.

Figure 2 | Transactions on the Secondary Bond Market in the CEMAC Zone



Source: Author's Calculations.

1.2. Bonds Auction

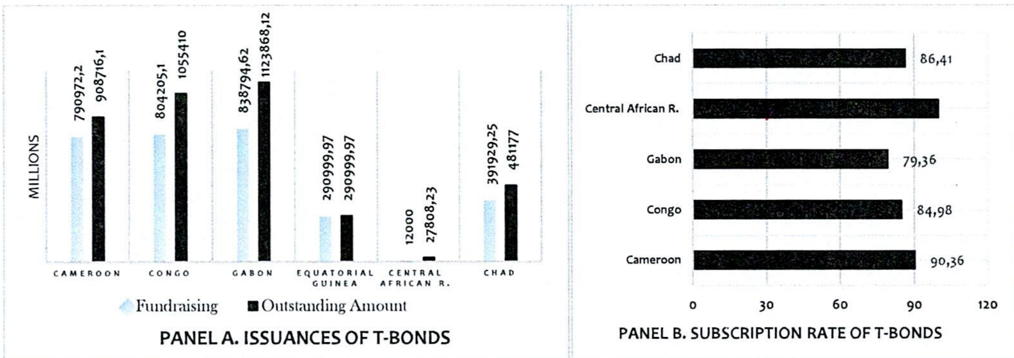
Bond auction or T-Bond or OTA is a fungible treasury bond which is dematerialized and registered at a custodian. OTA are negotiable securities issued for maturities greater or equal to two years, for a face value of XAF 10,000. They are monthly issued by each Treasury. Interests are annually paid and serve a bullet repayment to market participants. These long-term instruments are often used to finance investment expenses of member states. As in the regional stock market, bond government securities issued by auction comprise a primary and a secondary market.

1.2.1. Bonds Auction Issues

On the BEAC public securities market, Gabon has issued the highest stock of fungibles securities on the short- and long-term borrowing segments, followed by Chad and Cameroon for the short segment and Congo and Cameroon for the long-term segment (see Figure 3, Panel a). By contrast, the issuances of Cameroon are the most attractive in the sub-region securities market. Except for Cameroon, the subscription rarely reaches at 100 percent. The reasons vary across countries but the national funding policy can mainly explain why subscription rates are set this low level on the long-term compartment. More specifically, there is a possibility that the central bank repurchases the remainder not subscribed as soon as the tender specifications fixed by the agreement between BEAC and the national treasuries are fulfilled.

Figure 3 displays bonds issues through auctions among CEMAC countries.

Figure 3| T-Bonds Issues



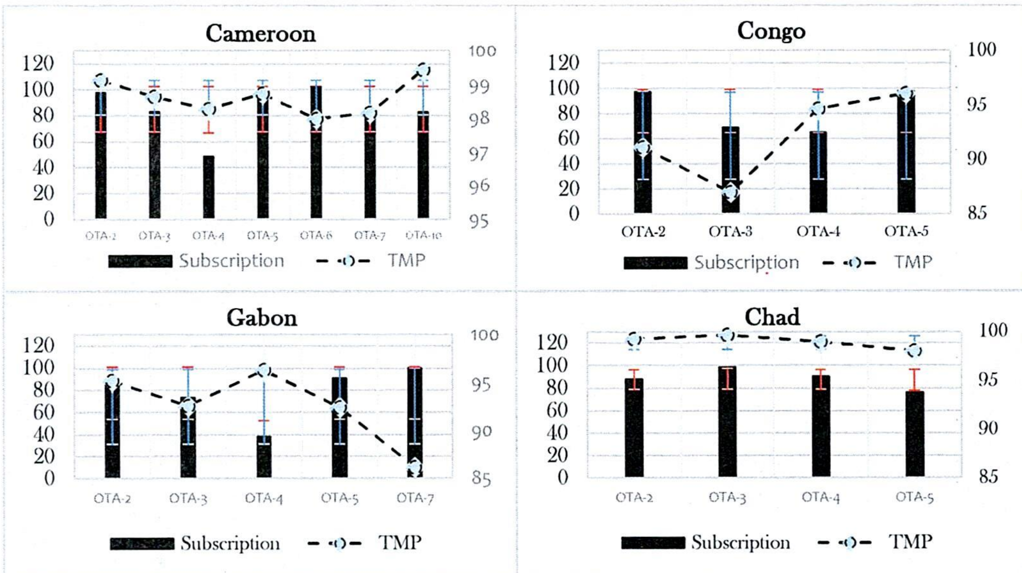
Source: CRCT/Author's calculations.

Gabon has issued a stock of XAF 1,123,868 million in the terms of fungible T-Bonds. Congo has also requested an important envelop of T-Bonds to sustain its public finances. The country is restructuring its public debt and has proceeded to a repurchase of 2-years maturity T-Bonds to reschedule its payments. All T-Bonds issues in Equatorial Guinea on the BEAC securities market were unsuccessful. The first attempt in March 2017 has been cancelled. The second tentative in February 2019 was a securitization which was not well received by investors. All OTA issues in Central African Republic were cancelled since the subscription rate does not reach 50 percent. The only exception was on May 7, 2021 when the country registered a 100 percent subscription rate.

Figure 4 reveals that Cameroon public securities are risky-free in the BEAC securities market so that they are used as diversification tools by investors looking for investment opportunities for their idle cash. Moreover, the main investors of the government securities issued by auction in the region are located in Cameroon. As a result, financing in Cameroon on the BEAC government securities markets is somewhat over-the-counter. Subscriptions to Cameroon T-Bonds are relatively low and very unstable across auctions. Bond discounts tend to decrease with maturity but the average weighted price does not vary significantly across auctions. In addition, Cameroon benefits to a risk-weighting derogation of its fungible securities by the Central African Banking Commission (COBAC).

Looking to Gabon, we can see that the regional market does not entirely absorb T-Bonds issues. The subscription rate to T-Bonds issues is often lower than 90 percent and the standard deviation across auctions is significant. Moreover, average weighted prices are often under 95 percent and bond discounts significantly decrease with maturity.

Figure 4| Yields and Investor Appetite for T-Bonds



Lecture: The graduation scale for the subscription rate is on the left and the graduation scale for the weighted average price is on the right. The standard deviation of the subscription rate is given by the red line and the standard deviation of weighted average price is blue line.
Source: CRCT/Author's calculations.

Equatorial Guinea suffers from three problems: the governance and the institutional framework, the low integration with other CEMAC countries and arrears due to local banks. Investors have a defiance against the governance and the judiciary system in Equatorial Guinea which do not provide all guarantees. The country is also the lesser integrated in the sub-region and the language is the main barrier which dampens the implication of non-resident primary dealers to auctions in Equatorial Guinea. Finally, the government has important arrears to local banks which are the main primary dealers in the country. This situation makes them risk-averse to auctions organized in Equatorial Guinea. In a different range, the government has important arrears to local bank which are the main primary dealers in the country. The country does not resort to T-Bonds since there is not imbalances on its public finances that undermine its public investments.

Central African Republic is the least active in the BEAC public securities. As a result, investors are not familiar with their T-Bonds issues. Chad is not active to many maturities, its issues concern 2, 3, 4 and 5 years with a subscription rate always greater than 78 percent. The Treasury also succeed in maintaining discounts under 6 percent on average.

Congo faces important problems in its public finances. The public debt exceeds 100 percent of its Gross Domestic Product (GDP) so that the International Monetary Fund (IMF) did not renew the access to the ECF. The country has proceeded to a restructuration of its public debt. As a result, the country raises funds on the BEAC government securities market at highest costs compared to other member states, excluding Equatorial Guinea. On the T-Bonds segment, the average weighted price displays significant variations across auctions.

Table 1 displays T-Bonds costs issues in the regional securities market.

Table 1 | T-Bonds Costs Issues in the Regional Securities Market

	Limit	Max.	Min.	PMP
Cameroon				
OTA-2	98.25	98.50	99.00	99.13
OTA-3	97.30	98.40	98.70	98.65
OTA-4	96.00	100.00	96.00	98.28
OTA-5	97.50	98.45	99.06	98.74
OTA-6	97.00	97.00	100.00	97.99
OTA-7	97.50	97.50	100.00	98.17
OTA-10	98.00	98.00	100.00	99.44
Congo				
OTA-2	90.34	90.52	94.56	91.17
OTA-3	91.78	91.78	95.05	87.04
OTA-4	94.67	94.67	95.00	94.71
OTA-5	96.11	96.11	98.11	96.12
Gabon				
OTA-2	91.08	92.76	94.02	95.24
OTA-3	91.08	91.26	94.23	92.62
OTA-4	92.63	92.63	96.50	96.35
OTA-5	89.57	89.57	94.46	92.46
OTA-7	85.00	85.00	97.00	86.18
Central African Republic				
OTA-3	88.00	88.00	100.00	92.21
Chad				
OTA-2	98.60	99.60	99.10	99.28
OTA-3	100.00	100.00	100.00	99.67
OTA-4	100.00	100.00	100.00	99.00
OTA-5	97.00	97.00	99.00	98.05

Source: CRCT/Author's Calculations

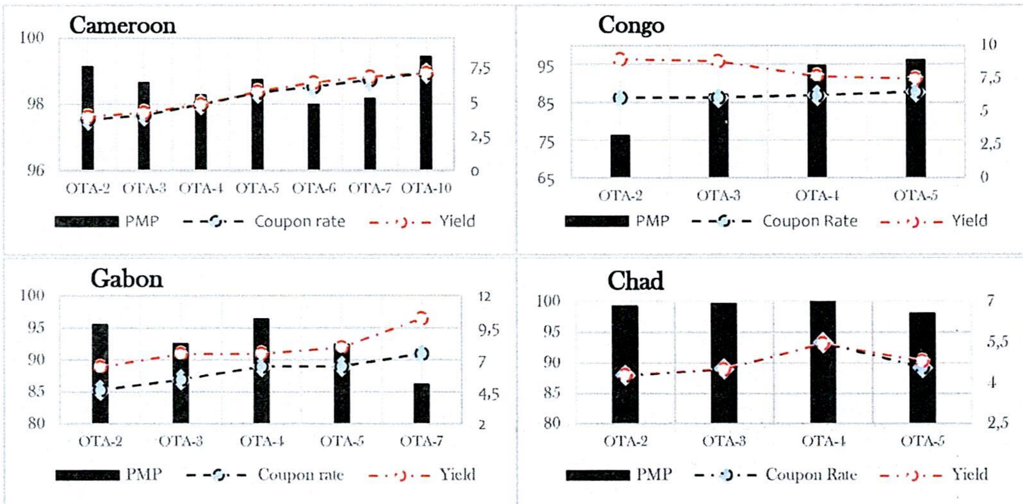
The cost of issues varies across member states. Prices offered by investors during auctions significantly vary in Gabon and Cameroon. The variations across auctions are more pronounced for long-term maturities than medium-term maturities. Table 1 shows that the difference between maximum price and minimum price for OTA-4 is about 4000 bps in these countries. Gabon registers the highest gap for the OTA-7. After an unsuccessful release on the BEAC securities market on December 16, 2020, the country has extended this 7-year maturity on December 23, 2020. The he divergent views of investors were linked to the first such issuance for Gabon, which was also facing a considerable drop in oil revenues.

The issuances on the long-term segment are also expensive for Congo which often retains the maximum price offered by investors during auctions. The country registers the lowest average weighted prices on average in the CEMAC zone. However, the T-Bonds discounts admitted by the government treasury during auctions are lower than in Gabon.

Figure 5 shows that T-Bonds discounts lead to high costs issues in the CEMAC zone. These costs are also aggravated by fees paid by national treasuries for the organization of each auction. As a result, the effective yields are significantly greater than coupon rate. T-Bonds issues in Congo and Gabon are attractive for investors. Not surprisingly, the effective yields to investors is most important in Gabon.

Cameroon and Chad maintain the effective yields sensitively equal to coupon rate. The Chadian Treasury does not retain limit weight that can deteriorate issue costs since the central bank can purchase the unsubscribed remainder according to its convention with national treasuries. By contrast, the low gap in Cameroon is due to the low risk of its government securities.

Figure 5 | Yields of T-Bonds Issues



Source: CRCT/Author's calculations.

1.2.2. Bonds Auction Transactions

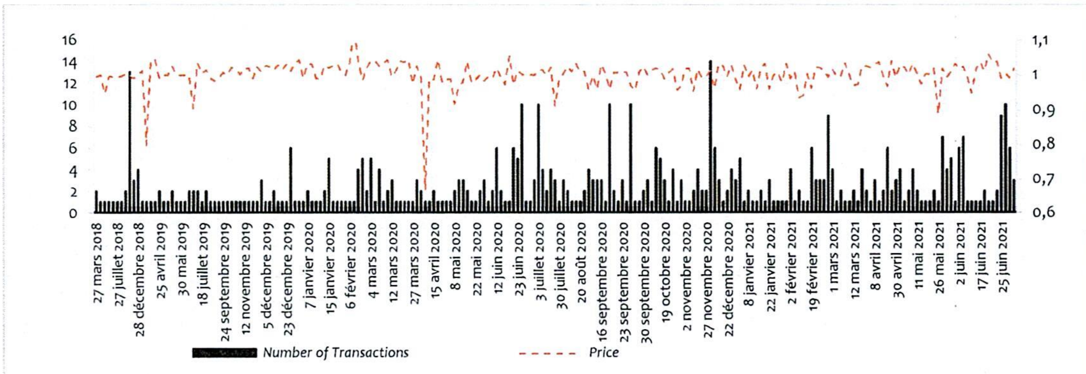
Primary dealers serve as market makers in the secondary by providing bid and ask quotes for T-Bonds. Transactions on this dealers market are run over the counter, investors execute orders separately and independently through dealers.

Gabon T-Bonds are the most liquid government securities in the regional market. The volume traded of Cameroon and Congo T-Bonds is also important. In contrast, the liquidity of government securities of Central African Republic, Chad, and Equatorial Guinea is weak.

Operations on the secondary markets are led by intra-group operations. This is not surprising insofar as the banking system of the monetary union is dominated by pan African banks (Attijariwa Bank, Ecobank, Afriland) and international banks (Société Générale, Citi, BGFI, etc). Since government securities are considered as investments, those banks have preferences for intra-group transactions since they allow a conservation of these securities in the books of parent banks. In addition, cash surpluses from other institutional investors (asset managers and insurance companies) are increasingly helping to boost secondary market operations.

Primary government securities dealers serve as intermediaries on the secondary market, receiving securities from sellers and delivering them to buyers. Figure 6 reveals that the volume traded on the secondary market is low and volatile. Even if the number of transactions does not exceed 15 to June 2021, a relative high activity has been recorded since May 2020. This effervescence did not bring prices uncertainty. We can see that see that the price of bonds traded vary between 99 and 101 percent. This suggest that a par yield curve is relevant for dealers which operate to trade government securities in the CEMAC.

Figure 6 | Primary Dealers Transactions (March 2018 – June 2021)



Lecture: The left axis refers to the number of transactions while the right axis is destined to the price at which the transaction was achieved.
Source: CRCT/Author's calculations.

1.3. Zero-Coupon Treasury Bonds

A zero-coupon bond, also known as an accrual bond, is a debt security that does not pay interest but instead trades at a deep discount, rendering a profit at maturity, when the bond is redeemed for its full face value.

Zero-coupon Treasury bonds (OTZ) are issued by signing securitization agreements in the Autonomous Sinking Fund (CAA). CAA serves as the central depositor. It ensures the centralization, conservation and circulation of scriptural securities for both issuers and intermediary account holders; the proper transmission of information relating to the life cycle of securities as well as that necessary for the settlement of authorised intermediaries, (sundry products: coupons, dividends, refunds, et.); and the unique coding of securities and issuers.

Funds raised are deposited in BEAC for the benefit of the Autonomous Sinking Fund. Their nominal value is XAF 1,000,000 CFA and their maturities vary from two to twelve years. The annual interest rate is 3 percent and the repayment is bullet. The settlement procedure or stripping takes place as soon as the CAA after checking the accounting of the buy and sell orders and the confirmation of operators.

The secondary market is an over-the-counter market and all transactions are run by a licensed operator (Treasury or licensed bank). Transactions are over-the-counter since securities are dematerialized and the transactions and communications are standardized.

Order n °00241/MINEFI /CAB /CAA December 01, 1995 defines the use of zero coupon bonds. Zero coupon Treasury bonds can be employed for the following uses: the guarantee of public contracts, the clearance of the liabilities of privatized companies vis-à-vis the Treasury, the payment of amounts owed by debtors to the Debt Collection Agency (SRC), the acquisition, in compliance with the legal provisions governing this matter of participations in the capital of companies to be privatized, the settlement of tax arrears prior to June 30, 1993, and the incorporation into the technical reserves of insurance companies.

2. Money Markets

Money-market instruments are short-term debt instruments with a maturity typically inferior or equal to 1 year. Some of these instruments such as certificates of deposit may have a maturity exceeding 1 year. These instruments are very sensitive to the monetary policy. There are basically three categories of issuers on this market: governments, banks and corporations. We will review T-Bills, negotiable debt securities and repos backed on government securities via subscription.

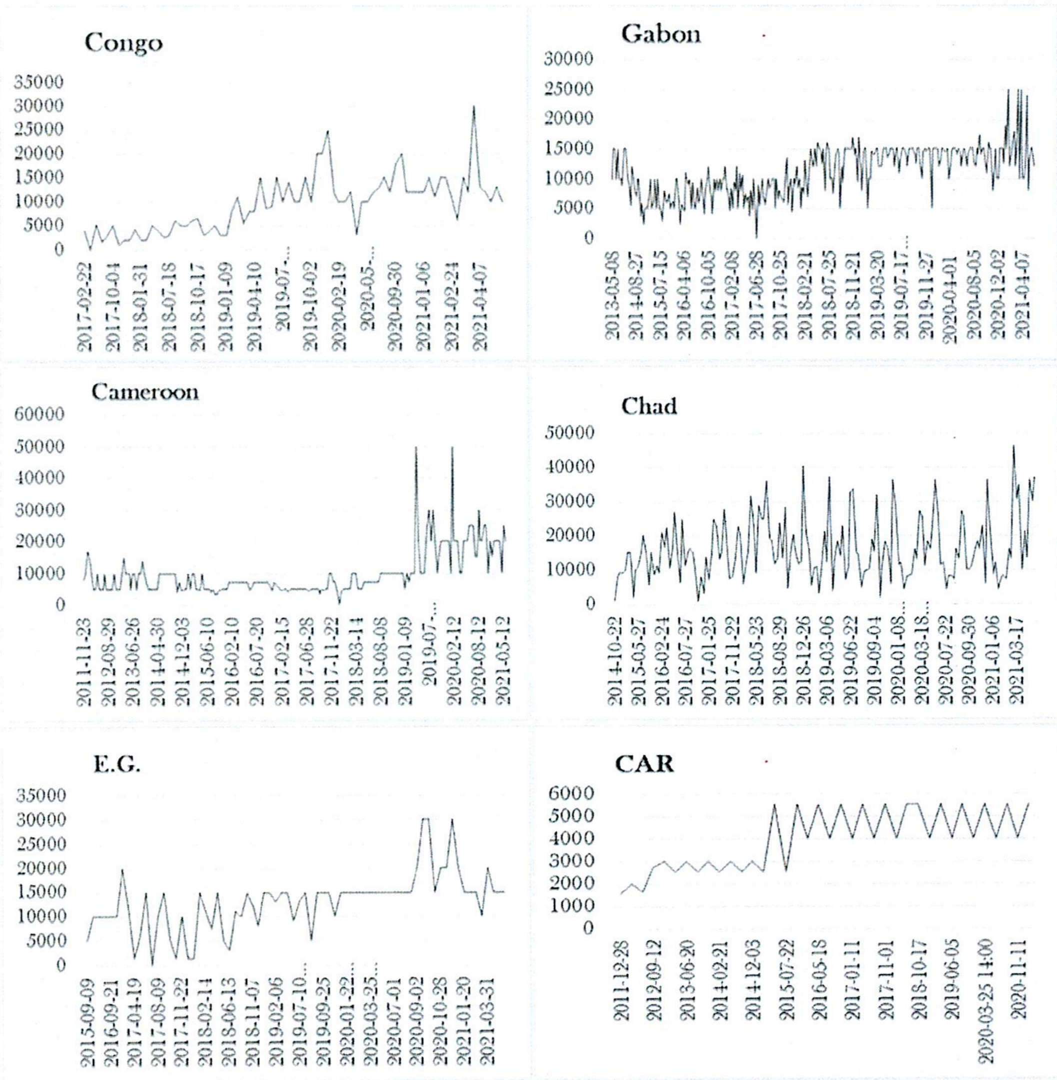
2.1. T-Bills

Treasury Bills are Treasury securities with a maturity below or equal to1 year. They entail no default risk because they are backed by the full faith and creditworthiness of the government. They bear no interest rate and are quoted using the yield on a discount basis on a money-market basis depending on the country considered. They are issued and traded on the regional government securities market as bonds auction. .

The six National Treasures can weekly issue T-Bills whose maturity is 13 weeks (BTA-13) or 3 months, 26 weeks (BTA-26) or 6 months, and 52 weeks (BTA-52) or 1 year. The nominal value is set at XAF 1,000,000 and the amounts are often low to insure that all the concomitant issuing of each member state at the same time is possible. As a result, each state have fifty-two issues by year. The BTAs are used to meet short-term cash flow needs caused by temporary mismatches between the fiscal revenue collections and the payments of operating expenses, especially salaries.

Figure 7 depicts the evolution of T-Bills in the CEMAC zone. We can see that amount issued at each auction in the short-term segment by Cameroon was relatively stable and low before March 2019. The persistence of insecurity and moderate oil prices have induced liquidity tensions so that the country has been obliged to increase credit lines in the BTA compartment.

Figure 7 | Stock of amounts issued in the BEAC government securities market



Souce : CRCT/Author's Calculations

T-Bills issues have significantly increased in 2019 in Congo. The country faced important treasury tensions and oil prices were moderate on international markets.

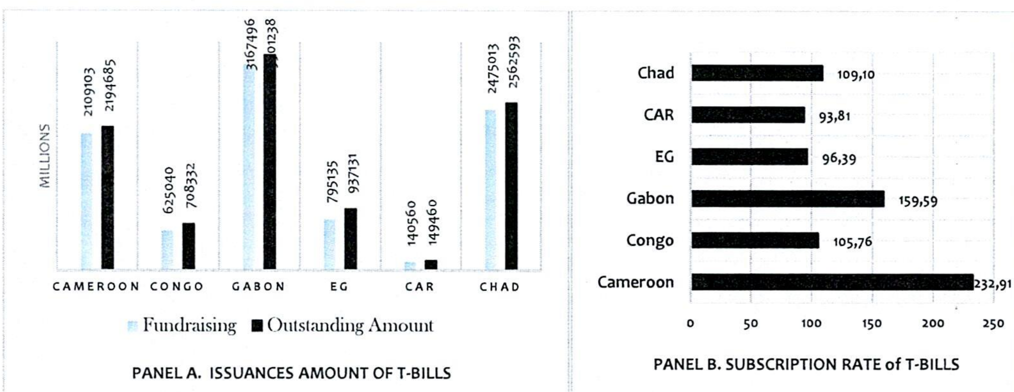
The amount of issuances in Gabon has significantly increased in 2021 since the conclusion of the Extended Credit Facility (ECF) of the three-year program 2021-2023 was not yet completed.

Until 2015, Equatorial Guinea did not resort to the regional public securities market. The collapse of international commodities markets and oil prices can explain why the country made it first period this year. Central African Republic does not significantly modify their issuances behavior. Although its first issue intervened before Equatorial, we can notice that the country has significantly raised its participation in 2015.

Except for two issuances through bank syndication in 2011 and 2013, Chad never resorted to the regional public securities market before the last quarter of 2014, when the country issued its first T-Bills. Since then, Chad has expanded its issuances and was among the largest issuer among CEMAC countries in 2015.

On June 4, 2021 the Gabonese Treasury has raised a stock of XAF 3,167,496 million fungible T-Bills in local currency (Figure 8, Panel a). The excessive use of short-term securities is associated to liquidity risks caused by permanent fiscal deficits and moderate oil prices observed after the collapse in mid-2014. The sensitivity of public finances in Chad to oil prices has also led this country to issue an important amount of T-Bills. Central Africa and Congo are the less active in the short-term segment the fundraising of Central Africa represents XAF 140,560 which represents 4 percent of the amount issued the Gabonese treasury. The weak activity of the Central African market can be explained by the atrophied network of primary dealers and the weak financial health of existing ones. Although, Gabon and Chad are the main animators of the short-term segment, we can see in Figure 8, Panel b that Cameroon remains the most demanded in the BEAC government securities market since the country presents the lowest risk in the region and its revenues do not extremely depend on oil exports as in Gabon and Chad.

Figure 8 | Government Issues in the primary market



PANEL A. ISSUANCES AMOUNT OF T-BILLS

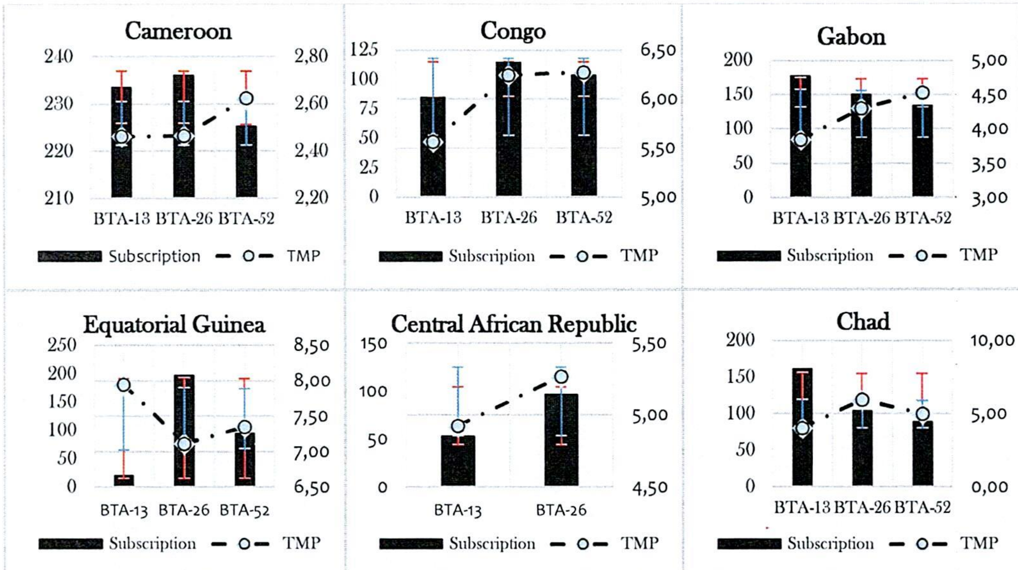
PANEL B. SUBSCRIPTION RATE of T-BILLS

Source: CRCT/Author's calculations.

Figure 9 depicts the risk appetite for T-Bills in the CEMAC zone. In Cameroon, investors operate trade-off in favor of BTA-26 to obtain a better yield. The BTA-52 does not has the same plebiscite but the average subscription overpasses 100 percent. It is also worth noting that the weighted average interest rate (TMP) varies in the same interval confidence.

Gabon government securities presents a good risk-return profile for investors. The subscription rate and the TMP evolve in opposite directions. However, the subscription rate overpasses 100 percent for each maturity issued in the short-term segment and the cost of issuance does not explode with when the maturity increases.

Figure 9 | Yields and Investor Appetite for T-Bills



Lecture: The graduation scale for the subscription rate is on the left and the graduation scale for the weighted average interest rate is on the right. The standard deviation of the subscription rate is given by the red line and the standard deviation of weighted average rate is blue line.
Source: CRCT/Author's calculations.

In Equatorial Guinea, market participants require high yields for very short-term maturities because the country records arrears. They mainly focus on the intermediary short term (BTA-26) to allow fiscal revenue collections. This behavior explains not only why the subscription rate is weakest for BTA-13 and the associated weighted average rate (TMP hereafter) is highest, but also why the subscription rate is highest for BTA-26 and the associated TMP is lowest.

CAR is the least active in the BEAC public securities. As a result, investors are not familiar with their issuance. This is the reason why the subscription rate is among the lowest in the region. The country does not resort to BTA-52 since investors. This could be explained by the fact that investors could require high yields given the weakness of its economy and the political instability which can increase its default probability for higher maturities in the short-term segment.

The subscription rate is highest for BTA-13 in Chad. The results is plausible with the fact that the country has already recorded a default on the BEAC government securities so that the subscription profile reflects the risk-aversion of investors. The low subscription rate observed for BTA-52 can also be caused by the fact that the investors are not yet familiar with this instrument. Chad does not use to issue this maturity. The first issuance has been recorded on January 2018.

Looking to Congo, we can see that investors are most interested by BTA-26 and require high yields since they are well informed about the debt unsustainability.

A clear hierarchy in terms of risk premiums exists among countries for 3-, 6-, and 12-month Treasury bills. Table xx displays the mean of the minimum rate, the maximum rate, the weighted average interest rate offered by primary dealers and the limit interest rate during auctions.

Table 2 | T-Bills Issues Costs in the Regional Securities Market

	Mean	Max	Min	Limit
Cameroon				
BTA-13	2.46	3.63	2.27	2.60
BTA-26	2.47	3.49	2.29	2.56
BTA-52	2.62	3.87	2.69	2.78
Congo				
BTA-13	5.57	6.26	4.89	5.96
BTA-26	6.24	6.40	5.70	6.40
BTA-52	6.27	6.38	5.84	6.38
Gabon				
BTA-13	3.84	4.45	3.65	4.10
BTA-26	4.30	4.95	4.03	4.50
BTA-52	4.53	5.21	4.20	4.79
Equatorial Guinea				
BTA-13	7.95	7.95	7.95	7.95
BTA-26	7.11	7.63	6.75	7.53
BTA-52	7.35	8.56	6.93	8.28
Central Africa Republic				
BTA-13	4.93			
BTA-26	5.27	5.42	5.16	5.21
Chad				
BTA-13	4.02	4.65	4.21	4.18
BTA-26	6.01	6.37	5.97	6.10
BTA-52	4.99	5.64	5.28	5.42

Source: CRCT/Author's Calculations.

Statistics show that Cameroon is the best risk in the zone. The TMP is 2.46 percent for BTA-13, 2.47 percent for 182 BTA-26, and 2.62 percent for BTA-52. The maximum rate offered by investors during auctions does not reach 4 percent in the short-term segment. The average limit rate for BTA-26 is smaller than the average limit rate for BTA-13. This surprising finding confirms the irrationality and the over-the-counter structure of Cameroon's financing on the BEAC securities market. The result is also in line with the high interest that investors have BTA-26 compared to BTA-13 although the earlier does not provide a remuneration which is significantly higher than the latter.

Gabon is the second best risk in the region. The country pays more than 120 basis points (bps) than in Cameroon for T-Bills segment. The outcomes of T-Bills issues are more intuitive than in Cameroon. More specifically the cost of issuances increases with the maturity. On average, the limit rate retained by the government after auctions is 4.10 percent for BTA-13, 4.50 percent for BTA-26, and 4.79 percent for BTA-52.

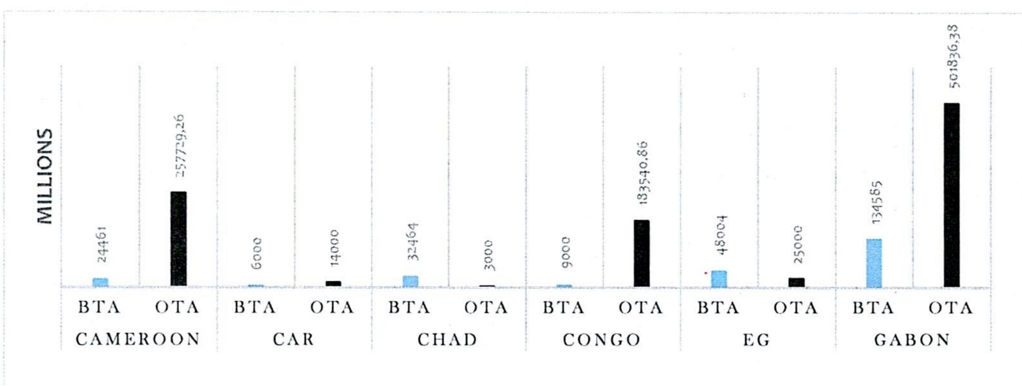
Central African Republic and Chad are at the third level in terms of risk in the CEMAC region. However, the reasons are not the same although the political instability is a common characteristic. Investors are unfamiliar with the Central African Republic market where Treasury does not usually resort to auctions. In the meanwhile, the idiosyncratic risk default is higher than in Cameroon and Gabon, this is why the interest rate are higher. The default registered by Chad can be the reasons why the country does not issue at the same cost that Gabon.

Due to its debt unsustainability and moderate oil prices, Congo faces significant costs to issue government securities in the short-term segment. Investors require between 4.89 and 6.26 on average for BTA-13, between 5.70 and 6.40 percent on average for BTA-26, and between 5.84 and 6.38 percent on average for BTA-52. The absence of other finance opportunities, especially from the International Monetary Fund (IMF), leads the government to retain the maximum rate offered for 6- and 12-monts T-Bills.

The cost of issuance is highest in Equatorial Guinea. This is not surprising and can be explained by the three factors mentioned in subsection 2.1. The country has paid more than 400 basis points (bps) than Cameroon in interest rates on 3- and 6-month Treasury bills.

Not surprisingly, T-Bonds are more traded than T-Bills on the secondary market of BEAC government securities (see figure 10 below). On the whole, investors adopt a hold and buy strategy for T-Bills since the repayment date is short and banks generally resort to repo markets when they face treasury shortages. Gabon T-Bonds are the most liquid government securities in the regional market. The volume traded of Cameroon and Congo T-Bonds is also important. In contrast, the liquidity of government securities of Central African Republic, Chad, and Equatorial Guinea is weak.

Figure 10 | Volume traded of government securities in the BEAC secondary market



Source: Author's Calculations.

2.2. Negotiable debt Securities

Regulation n°04/08/CEMAC/CM enacted in March 27, 2016 distinguishes: certificates of deposit (CD) and commercial papers. This regulatory act states that these money instruments have an initial maturity lower or equal to two years. Their facial value is XAF 1,000,000 or a multiple of this amount and may be issued at a price other than par under the condition that the issuer publishes the annual yield at the end of the year.

Certificates of deposit are offered by banks and deposit and consignment offices that provides an interest rate premium in exchange for the customer agreeing to leave a lump-sum deposit untouched for a predetermined period of time. Commercial papers are commonly used type of unsecured, short-term debt instruments issued by non-credit institutions including corporations, development finance and guarantee bodies, public firms, national insurance funds, and member States.

The licensed issuers mentioned above are approved by the central bank following an issuance program and a documentation relating to the activity and financial statements of applicants. The documentation and its updates must be available to public at the issuer's headquarter, securities domiciliary bank, and on the BEAC website. Only operators established in CEMAC can issue negotiable debt securities.

In addition to the issuer, only the following are authorized to place or negotiate negotiable debt securities: credit institutions, deposit and consignment offices and licensed brokers. Issuers can buy and conserve negotiable debt securities that they have issued within the limit of 10 percent of the outstanding amount of each issuance program.

The interest rate is freely fixed by the issuer. The interest rate is fixed and prepaid for issues with an initial maturity lower or equal to one year. When the maturity is greater than one year, the interest rate can be variable and anchored to a reference money market rate. The interest rate is then annually payable at maturity.

Even if the regulatory framework exists, the monetary union has not registered an issue of negotiable debts securities.

2.2.1. Repo and repo reverse markets

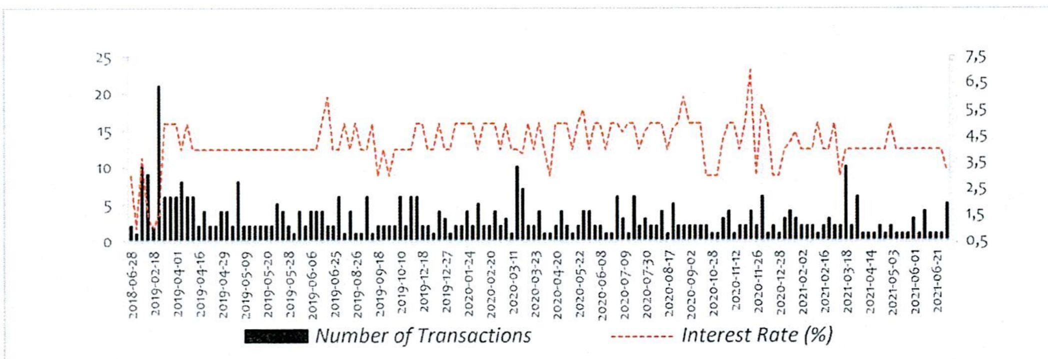
A repurchase (repo) is a mechanism by which an investor lends bonds in exchange for a loan of money, while a repurchase reverse (repo reverse) is a means for an investor to lend money in exchange for a loan of securities. Regulation n°03/CEMAC/UMAC/CM enacted in March 27, 2015 set the regulatory framework of repo and repo reverse operations on the money market of the CEMAC of the CEMAC zone.

Financial assets susceptible to candidate for repo or repo reverse transactions are: stocks issued by CEMAC member states or abroad, Treasury bills and bonds, negotiable debt securities issued in one of the CEMAC member states as well as similar instruments issued abroad, public and private non-negotiable instruments, and all debts represented by a negotiable instrument on a market.

Repo transactions are concluded with or without prior creation of a margin. When the repo is concluded with a margin, parties agree on the methods for margins fixing and retrocession. When there is a prior creation of a margin, the parties determine financial assets or additional cash contribution that may be necessary to take variations into account.

On the secondary market, repo operations in the primary dealers system are more important than volume traded on the BEAC securities market and interest rates often vary from 3.5 to 5.5 percent (see Figure 11).

Figure 11 | Reo and Reverse Repo Transactions (March 2018 – June 2021)



Source: CRCT/Author's calculations.

Figure 12 displays a breakdown of primary dealers reverse repo positions by tenor. We can notice that financing needs vary across CEMAC countries, leading to demand for loans of a different tenor.

Overnight-tenors have been abandoned in Cameroon and Congo. In Cameroon, investors have substituted overnight repos by tenors lower than 30 days which cost 4.5 percent on average with a volatility of 100 bps across operations.

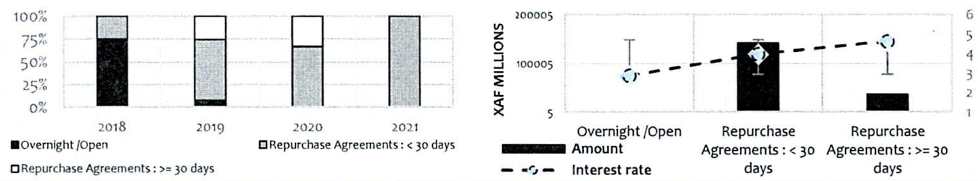
In Congo, primary dealers' financing needs usually overpass 360 days. Dealers have progressively abandoned overnight repos and repurchase agreements lower than 30 days.

Overnight tenors are mainly preferred by primary dealers for Chad and Central African Republic securities. However, some repo transactions for tenors lower than 30 days are registered in the primary dealers system of the Central African Republic.

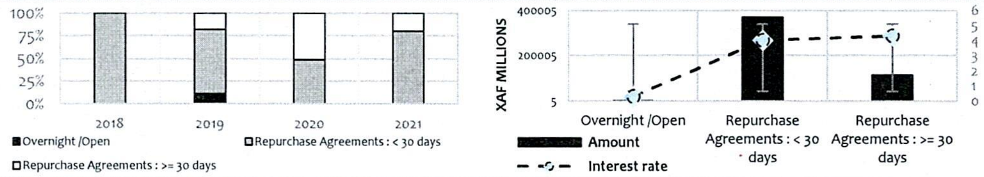
By contrast, they are not employed by primary dealers for Gabon securities since they do not provide high yields. More generally, their costs are the smallest than in the CEMAC zone.

Figure 12 | Primary Dealers Reverse Repo Markets

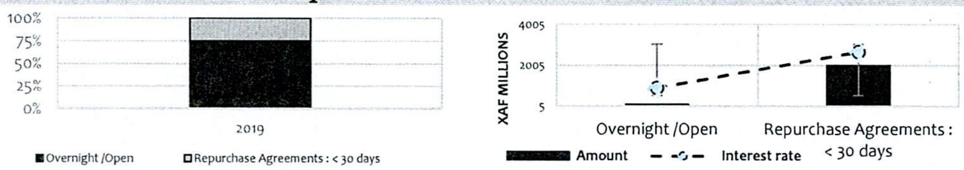
Panel A. Cameroon



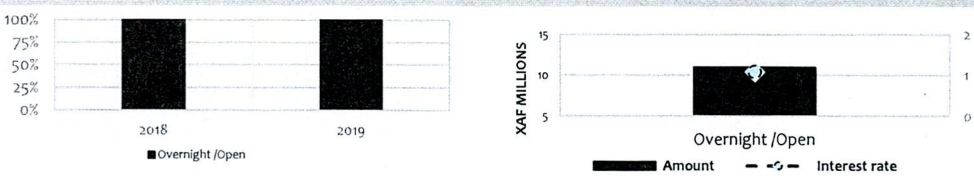
Panel B. Gabon



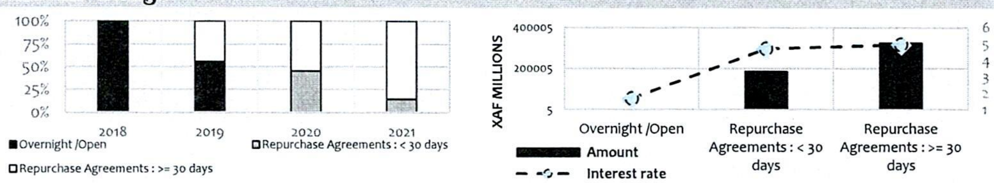
Panel C. Central African Republic



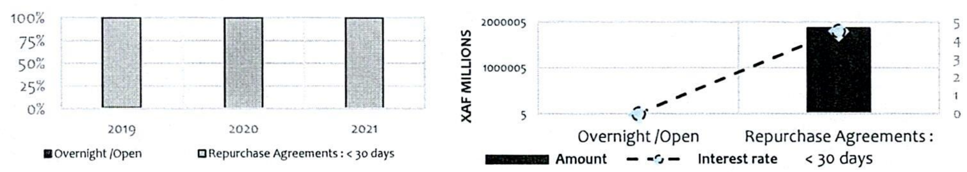
Panel D. Chad



Panel E. Congo



Panel F. Equatorial Guinea



Source: CRCT/Author's Calculations.

The main goal of this chapter was to present the investment environment of fixed income securities in the CEMAC zone. A review shows that there are two main classes of fixed income instruments: bonds and money instruments.

Bonds cam be gathered in three categories: bond syndication, bonds auction, and zero-coupon Treasury bonds. Bonds syndication are served to investors through a private placement or a public offering. In the CEMAC zone, bond syndication through public offering are dominated by sovereigns. Corporate issues are limited by four main problems: the weakness of the banking system, a rudimentary payment system, the credibility of issuers, and the unreliable structured information on markets. Moreover, there are a dual taxation system in the CEMAC zone that exempts sovereign issues while corporate issues are taxed.

Bonds auction have been instituted to avoid the direct financing of member states by the central bank. Transactions costs are less expensive than bonds syndication. Except for Equatorial Guinea, all CEMAC countries have already resorted to T-Bonds. Gabon is the most active Treasury for these securities in the monetary union. T-Bonds issues offer different yields across countries. Congo faces the highest costs because of the unsustainability of its public finances.

Zero-coupon Treasury bonds (OTZ) are issued by signing securitization agreements in the Autonomous Sinking Fund (CAA). The secondary market is an over-the-counter market and all transactions are run by a licensed operator (Treasury or licensed bank). Transactions are over-the-counter since securities are dematerialized and the transactions and communications are standardized.

Looking to money markets, three types of fixed income instruments exist in the Zone: T-Bills, negotiable debt securities, and repos. T-Bills are the most prominent securities in the money market. CEMAC countries deliver 13-, 26-, and 52-weeks maturities. As for bonds auction, the subscription rate and their issue costs vary across countries. Other money instruments such as CD and commercial papers are not yet issued in the monetary union. Repo and repo reverse operation supported by government debt securities are expanded. They offer the same yield regardless the security on which they are backed.

In the next chapter, we will present the theoretical background of yield curve which is at the center of fixed income markets. We will review traditional term structure theories and the main models of the yield curve.

Chapter II | Background of Yield Curve and Bond

From the mathematical point of view, the term structure also known as the yield curve is equivalent to estimate the spot rates, the forward rates or the discount function [see for instance Deacon and Derry (1994)]. More specifically, it is a graph that plots the yields of similar quality bonds in the vertical axis with the corresponding maturities, or time, in the horizontal axis. The term structure reflects expectations of fixed income market analyst about future changes in interest rates as a result the shape of the curve helps investors to anticipate the likely future course of interest rates. A fixed income analyst can also determine whether and the extent to which securities are overpriced or underpriced examining the position of its yield compared with the yield curve.

Many macroeconomic factors can drive influence and explains its shape and its behavior across time. An economic analysis leads to suggest inflation as the most direct determinant since central banks tend to respond to an increase of expected inflation with an increase in interest rates. Moreover, good economic outlook may lead to inflationary upwards due to a rise in aggregate demand. In a different range, long savings plans, the aging of the population and uncertainties about expectations can have a significant impact on the evolution of long rates. All these factors drive the shape of the term structure.

The present chapter investigates the theoretical background behind the yield. In practice, the term structure has different shape restrictions which are based on different economic theories which suggest that interest rates are led by expectations [e.g. Bodie et al. (1996) and Douglas (1988)], liquidity preference [for instance Douglas (1988) and Fisch (1997)], market segmentation [see Culbertson (1957)] and preferred habitat. Different approaches for fitting and estimating the yield curve using regression techniques, splines, parsimonious and stochastic processes. The chapter also present the theoretical models used for fitting and estimating yield curves.

The rest of the chapter is straightforward. In Section 1, we present the main theories which explain different shapes that the yield curve can take. For that purpose, we present pure expectations hypothesis, liquidity preference hypothesis, market segmentation hypothesis and the preferred habitat theory. The second section reviews models for fitting and modelling yield curve, namely regression, empirical and equilibrium models.

1. Yield Curve Theories

In this section, we review traditional economic theories of the yield curve that attempt to explain the relationship between interest rates and their residual maturity. They can be gathered in four categories: the pure expectations theory; the liquidity premium theory, the market segmentation theory and the preferred habitat theory.

1.1. Pure Expectations Theory

The expectations theory defends that investors' expectations is the main determinant of future interest rates. This theory suggests that investors require a premium to possess fixed income securities which maturity is different from their investment horizons. Put in a different way, the expectations theory highly relies on the assumption that investors are indifferent to tenors as long as they register the highest return over the investment period [see for instance Bodie et al. (1996) and Douglas (1988)]. There are two main versions of this theory: the local expectations hypothesis and the unbiased expectations hypothesis. The expectations theory is also discussed through the return to maturity expectations hypothesis and yield to maturity expectations hypothesis [see Ingersoll (1987)]. In this subsection, we only discuss the distinction between local and unbiased expectations.

1.1.1. Local Expectations Hypothesis

The local expectations hypothesis has a simplistic approach to examine views that investors can have on securities. It suggests that bonds with the same quality, but different in maturities, have the same expected holding period rate of return. This suggests for example that a six-month bond and a twenty-year bond will produce the same rate of return, on average, over the stated holding period. Within this context, investors are not risk averse and do not necessarily require a reward for taking on higher risk.

Moreover, this hypothesis does not provide an interesting issue about the shape of the term structure. Cox et al. (1981) has shown that the local expectations argument is most appropriate to derive the equilibrium between spot and forward rates. It is an economic approach which does not take other determinants that impact on holding period returns between bonds that do not have similar maturities. For instance, investors face requirements on the choice of bonds they can hold. For instance, banks are required to possess short-dated bonds for liquidity purposes. As a result, these investors will have to hold shorter-dated bonds, even if the associated return is lower.

Consequently, although it is economically admitted to expect that the return on a long-dated securities is equivalent to rolling over a series of shorter-dated bonds, it is usually observed that longer-term returns exceed annualized short-term returns. So, an investor that mainly take active positions in short-dated zero-coupon bonds are supposed to register a lower return than if they had invested in a long-dated zero-coupon bond. Rubinstein (1999) gives a straightforward explanation of this phenomena. The author suggests that compared to theoretical models, future spot rates are not known with certainty in practice. Short-dated zero-coupon bonds are more attractive to fixed income markets investors for two reasons. First, they are more appropriate instruments for hedging purposes. Second, they are more liquid instruments, being readily converted into cash than long-dated instruments. Therefore, the demand for short dated maturities is often higher, and hence short-term yields are often lower than long-term returns.

1.1.2. Pure or Unbiased Expectations Theory

The pioneer discussion of pure expectations was proposed by Lutz (1940) and Fisher (1896). This theory assumes that current implied forward rates are fair estimators of future spot interest rates. The investors operate such that there is no advantage of holding instruments of a particular maturity. A positively sloping term structure leads to a rise of spot rates while an inverted term structure suggests that spot rates decrease. As a result, if the short rates rises, then longer rates must be higher than shorter to reflect this assumption. If short rates are higher than longer rates, investors would cetiris paribus buy shorter-dated bonds and renew investment at maturity. In the same vein, if the market expects that rates fall, then longer yields should be lower than short yields.

The unbiased expectations hypothesis states that the long-term interest rate is a geometric mean of expected short-term rates. Technically, we can derive the forward yield curve using the following equations:

$$(1+s_N)^N = (1+s_1)(1+{}_1f_2)\cdots(1+{}_{N-1}f_N) \tag{1}$$

or:

$$(1+s_N)^N = (1+s_{N-1})^{N-1}(1+{}_{N-1}f_N) \tag{2}$$

where s_N is the spot yield on a $N-$year bond and ${}_{n-1}f_n$ is the implied one-year rate n years ahead.

This result is important to ensure no arbitrage opportunities in the market. It suggests that the forward rate equals the subsequent spot rate. Therefore, the unbiased expectations hypothesis considers that ${}_0f_2$ is an unbiased estimator of the spot rate ${}_1s_1$ observed one period later. The unbiased expectations hypothesis explain any shape of the yield curve. A rising yield curve is therefore caused by expectations that short term interest rates, ${}_1f_2 > s_2$. Inversely, a falling term structure is caused by investors expecting short-term rates to be lower in the future. A humped yield curve is explained by investors which expect short-term interest rates to rise and long-term rates to fall.

Expectations on interest rate dynamics are functionally linked to expectations on inflation. If investors expect inflationary upwards, the term structure will likely be positively shaped whereas if expected inflation is inclined towards downtrend, the term structure is supposed to be negative. However, several empirical researches including the paper of Fama

(1976) have shown that forward rates are essentially biased predictors of future spot interest rates, and often overestimate future levels of spot rates. The unbiased theory is also unrealistic for suggesting that investors can forecast very long-dated spot interest rates.

1.2. Liquidity Preference Theory

The liquidity preference theory differs from the expectations theory in the way that the notion that investors are indifferent to the investment maturities is rejected. It can be intuitively admitted that longer maturity investments are more risky than shorter ones. More specifically, investors prefer short-term horizon because long-term horizon is associated to higher interest rate risk. This theory therefore postulates that investors compensate with a higher return on long-term investments. For example, a financial institution which lend money for a five-year term will often demand a higher interest rate than for a five-week term. The reason is that borrower can record a default over the longer time period as they may for instance, have gone bankrupt in that period. As a result, longer-dated yields should be higher than short-dated yields, to compensate the lender for the higher risk exposure during the term of the loan. The liquidity premium also relies on the fact that greater demand for short-term securities increases the liquidity of the instruments, which means their yields are lower than those of long-term securities. The yield curve will be upward sloping when short-term securities have lower yields than long-term securities [see Douglas (1988) and Fisch (1997)].

Borrowers often prefer to borrow over as long a term as possible, while lenders will wish to lend over as short a term as possible. Therefore, as we first indicated, financial institutions have to be compensated for lending over the longer term. This mechanism is considered as a premium for a loss in liquidity for the lender. The premium is increased the further the investor lends across the term structure, so that the longest-dated investments will, cetiris paribus, have the highest yield. So the liquidity preference theory states that the term structure should almost always be upward-sloping because of bondholders' preference for the liquidity and lower risk of shorter-dated bonds. An inverted yield curve could still be explained by the liquidity preference when t is combined with the unbiased expectations hypothesis. A humped yield curve might be viewed as a combination of an inverted yield curve together with a positively sloping liquidity preference curve.

However, the difference between a term structure explained by unbiased expectations and an observed term structure sometimes refers to the liquidity premium. This can be explained by the fact that short dated maturities are more liquid than long dated maturities although it is not easy to measure this phenomena. Indeed, liquidity premium is dynamic and changes with the quality of bonds.

The setting of a liquidity premium aims at encouraging investors to hold long dated securities, the yields on such instruments must be higher than short-dated securities which may be converted into cash more easily. The liquidity premium is that fixed income investors require for holding less liquid maturities. If long dated instruments then provide higher yields, they should generate on average, higher total returns over an investment period. This is not consistent with the local expectations hypothesis. More formally we can write:

$$0 = L_1 < L_2 < L_3 < \cdots < L_n$$

and

$$(L_2 - L_1) > (L_3 - L_2) > \cdots < (L_n - L_{n-1})$$

where L_n is the premium for a bond with n years maturity. This relation suggests that the premium increases with the term to maturity. It can also reflects the fact that an otherwise flat yield curve will have a positively sloping curve, with the degree of slope steadily decreasing as we extend along the yield curve.

1.3. Market-segmentation theory

Fixed income markets are constituted by of a wide variety of agents, each with different investment horizons. Some investors have preferences for the short part of the term structure, while others focus on the longer part. The segmented markets hypothesis suggests that investors have preferences for securities that match their needs and separate demand and supply determinants exists for short-term and long-term securities and the shape of the yield curve is primarily determined by the market law and interactions of investors [see Stander (2005)]. More specifically, activity is concentrated in certain specific areas of the market, and there are no spillovers among these parts of the market. The relative amounts of funds invested in each of the maturity segment produces differentials in supply and demand, resulting in humps in the term structure. Put in other words, the shape of the yield curve is determined by supply and demand for certain specific maturity horizons, each of which has no reference to any other part of the yield curve.

Considering two classes of investors: the first group buys short-term securities while the second buys long-term securities. The two groups may influence the prices of the investment instruments on which they are operating, since they instantaneously increase the demand for the securities and also reduce their effective liquidity by holding on to the securities. The interrelationship of the two groups thus creates an interest rate differential between securities with different maturities [see Bodie et al. (1996)]. Choudhry (2019) supports that financial institutions such as banks and building societies concentrate a large part of their activity at the short part of the term structure for daily cash management (known as asset and liability management) and for regulatory purposes (known as liquidity requirements). Fund managers such as pension funds and insurance firms are most active at the long part of the term structure. Few institutional investors, however, have any preference for medium-dated bonds. These different behaviors of investors lead to high prices (low yields) at both the short and long ends of the yield curve and lower prices (higher yields) in the middle of the term structure.

According to the segmented markets hypothesis, a separate market exists for specific maturities along the yield curve, and interest rates for these maturities are determined by the supply and demand law. This suggest that the yield of fixed income security lies above other segments when there is no demand. Investors do not hold securities in any other area of the term structure outside their area of interest. As a result short-dated and long-dated yields exist independently of each other. The segmented markets theory is often illustrated by reference to banks and life insurance companies. Banks and building societies hold their funds in short-dated securities, usually no greater than five years maturity. This can be explained by the nature of banking operations, with a large amount of funds being deposited at banks, and also for regulatory purposes. Holding short dated, liquid maturities allows banks to face any sudden or unexpected demand for funds from depositors. Insofar as banks invest their funds in short-term instruments, the required yields are driven down. This affects the short part of the term structure, but not the long end.

The segmented markets theory can be exploited to explain any shape of the yield curve, although it fits best perhaps with positively sloping yield curves. However, it offers no information content during analysis since it cannot be used to interpret the yield curve whatever shape it may be. The theory suggests that for investors, instruments with different maturities are not perfect substitutes for each other since they have different holding period returns. As a result of bonds being imperfect substitutes, markets are segmented according to maturity. The segmentations hypothesis is an acceptable explanation of several features of a conventional positively sloping yield curve, but by itself is not sufficient. It is recognized that banks and must hold short dated securities for regulatory purposes and yield considerations. However, other investors are probably more flexible and will place funds where value is deemed to exist. Nonetheless the higher demand for benchmark securities drives down yields along certain segments of the yield curve.

1.4. Preferred habitat theory

The preferred habitat theory is s slightly modified version of the market segmentation hypothesis [see Choudhry (2019)] while Martellini et al. (2003) defend that it is an improvement of the liquidity premium.

The preferred habitat theory suggests that different fixed income investors have an interest in specified parts of the term structure, but can be encouraged to hold bonds from other areas of the maturity spectrum if there are sufficient incentives. From this assumption, banks may at certain times hold long-term securities once the price of these bonds falls to a certain level, making the return on the bonds worth the risk involved in holding them. The same considerations may persuade life insurance companies to hold short-dated maturities. Put in a different way, higher yields will likely be required to make bond holders shift out of their usual area of interest. The preferred habitat hypothesis recognizes the flexibility that investors have, apart from regulatory requirements, to invest in whatever area of the term structure when they identify value.

Martellini et al. (2003) provides another interpretation of the preferred habitat theory which is an extension of the preference liquidity because it postulates that risk premium is not uniformly increasing. Indeed, investors do not all intend to liquidate their investment at any period of time since their investment horizon is dictated by the nature of their liabilities. Nevertheless, when bond supply and demand on a specific area of the term structure do not match, market participants are ready to move to other curve parts where there is the inverse disequilibrium insofar as they receive a risk premium that offsets their price or reinvestment risk aversion.

2. Overview of Yield Curve Models

The yield curve is an important tool for market participants, market authorities and academic researchers. It allows practitioners to price any fixed-income security offering known cash flows in the future and to obtain implied curves [see Martellini et al. (2003)]. Deriving such a curve has led researchers to build different theoretical models. Stander (2005) has provided an extended overview of the main types of yield curve models including regression-type models, empirical models, and equilibrium models. The following discussion heavily draws on its survey.

2.1. Regression-Type Models

The conventional way of constructing a yield curve relies on observed prices according to maturity for a series of bonds. Then, the yield curve is the function that fit optimally the data. Technically, linear and nonlinear regression methods are employed to adjust a scatter plot of yields. The basic way consists to minimize the squared differences between observed yields and fitted yields as follows:

$$\hat{\beta} = \arg\min_{\beta} \left[\sum_{i=1}^{n} \left(r_i - \hat{r}_i \right)^2 \right] \tag{1}$$

where r_i corresponds to observed yield to maturity of bond i, $\hat{r}_i$ is the fitted yield to maturity of bond i with the specified model, and n the number of bonds used to construct the curve. There are various regression-type models to derive the yield curve [see for instance Bradley and Crane, Elliot and Echols, Dobbie and Wilkie (1979)].

The Bradley-Crane model is specified as in the following equation:

$$\ln\left(1 + r_i\right) = \beta_0 + \beta_1 t_i + \beta_2 \ln\left(t_i\right) + \varepsilon_i \tag{2}$$

where $\beta_i, i = 0,1,2$ are parameters to be estimated, t_i is the term to maturity i, and $t_i \neq 0$. ε is the term error, $\varepsilon \sim N\left(0, \sigma^2\right)$.

Elliot and Echols attempt to capture the effect that coupons have on the yields to maturity. They propose a three-dimensional yield curve which is specified as follows:

$$\ln\left(1 + r_i\right) = \beta_0 + \beta_1 \frac{1}{t_i} + \beta_2 t_i + \beta_3 c_i + \varepsilon_i \tag{3}$$

with c_i the coupon rate.

These models are not flexible since they cannot fit in a good way different types of shapes taken by a yield curve.

The Dobbie and Wilkie's (1979) model has the following specification:

$$r_i = \beta_0 + \beta_1 e^{-\alpha_1 t_i} + \beta_2 e^{-\alpha_2 t_i} + \varepsilon_i \tag{4}$$

where $\alpha_i, i = 1,2$ are non-linear regression coefficients. This functional specification was employed by several practitioners. However, the fitted values can lead to significant jumps [see Cairns (1998), Feldman et al. (1998), and Stander (2005)].

The Ayres and Barry's model improves the fitting. However, its theoretical specification is not as malleable as the Dobbie-Wilkie model and displays the same limitations that Elliot and Echols and Bradley and Crane. The functional form is given by the following equation:

$$r_i = r_\infty + e^{-\beta(t_i - t_0)}\left(r_0 - r_\infty\right) \tag{5}$$

where r_∞ is the yield on a perpetuity or long-term instrument, β a fitted coefficient, and r_0 and t_0 the yield and the term of the shortest security in the sample.

The Super-Bell model presented by Bolder and Stréliski (1999) presents a mathematical framework for a par curve:

$$r_i = \beta_0 + \beta_1 t_i + \beta_2 t_i^2 + \beta_3 t_i^3 + \beta_4 \sqrt{t_i} + \beta_5 \ln\left(t_i\right) + \beta_6 C_i + \beta_7 C_i t_i + \varepsilon_i \tag{6}$$

with the same notations as above.

The fitting of the model is achieved through two steps. First, the parameters are estimated using the least squares. Second, the fitted par yields are used in another regression where the coefficients are estimated again. However, the parts depended on the coupon rate are removed. This model presents the main limitation that the zero-coupon curve only exists for discrete points. This means that some supplementary assumptions are needed to extrapolate between discrete points [(Stander (2005)]. It can also lead to strange shapes of forward rate curves and does not always adjust latent bonds suitably [see Bolder and Stréliski (1999)].

McLeod (1990) has proposed a methodology based on the cluster analysis to derive de yield curve. Its theoretical framework tries to address the issue where different bonds with the same term to maturity do not necessarily have the same yield to maturity. Technically, the bonds are organized into different groups according to their terms to maturity using selected knot point and a cubic spline function is then fitted through these points. The main shortcoming with this approach is that the derived yield curve relies on the chosen knot points. As a result, the individual par yields may not be fitted well. Furthermore, the cubic spline does not allow the convergence to a constant level in the long term.

The regression-type models present many problems so that they cannot be considered or used in practice to derive yield curves. First, their simplified specifications of yield curves do not allow an adequate fit of observations. Second, the use of ordinary least squares could lead to very volatile parameters since the associated assumptions are not necessarily met. Finally, the regression-type models rely on the underlying idea that when fitting a curve through the yields to maturity, there is not an explicit restriction of payments due on the same date to be discounted at the same rate [see Anderson et al. (1997)].

2.2. Empirical Yield Curve Models

The empirical models try to overcome some of the shortcomings of the regression-type models. Empirical yield curve models derive the yield curves fitting observed marked data. They rely on mathematical discount function and estimate the parameters under the constraint that when each of the bond's cash flows are discounted with it, the adjusted bond price equalizes the current market price.

The price, P_i, of a default-free bond $i = 1,...,N$ can be expressed as a linear combination of promised cash flows as follows:

$$P_i = \sum_{j=1}^{n_i} C_{i,j} df_{m_{i,j}} \tag{7}$$

where n_i is the number of outstanding cash flows of bond i, N is the number of bonds used to construct the yield curves, df_m is the discount function for a maturity m, $m_{i,j}$ is the maturity in years from the value date to the $j-th$ future cash flow, $C_{i,j}$ of bond i.

2.2.1. Polynomial and Spline Methods

The Weierstrass approximation theorem indicates that any continuous function over an interval with a random small degree of error can be uniformly approximated as by a class of functions. As a result, a continuous yields curve is susceptible to be adjusted, with an arbitrary accuracy, by a polynomial function. This is the most basic version of empirical models to derive yield curves. The model can be written as:

$$s(m) = \sum_{j=1}^{J} \alpha_j m^{j-1} \tag{8}$$

where $s(m)$ is the spot rate with a maturity m, α_j is the $j-th$ polynomial coefficient, and J is the length of polynomial.

Under the assumptions of continuous compounding (see Chambers et al. 1984), the discount function of the exponential model is written as:

$$df_m = \exp\left\{-\sum_{j=1}^{k} \alpha_j m^j\right\} \tag{9}$$

where α's parameters are estimated and k is the degree of the polynomial. Chambers et al. (1984) suggest that a third-degree polynomial is often sufficient.

The polynomial models are simplistic approaches to derive yield curves. However, their use introduce significant shortcomings: (i) it does not have the ability to give a high weight to values of m which are more likely to occur, (ii) the model can lead to extreme values between observation on the run since it requires an extremely high-order polynomial to fit both the long and short end of the curve, (iii) McCulloch (1971) and Coleman et al. (1992) support that the polynomial function appears to fit strongly data for long term maturity while the smoothing over shapes implied by data at the near end, (iv) the polynomial is not monotonically decreasing and this is does not match a requirement of a discount function.

Several functional specifications have been developed to avoid the shortcomings of a simple polynomial modeling when smoothing the term structure. Schaefer (1981) employs a Bernstein polynomial methodology to estimate term structure of interest rates while McCulloch (1975) proposes a spline method to construct the yield curve.

McCulloch (1975) proposes a simple cubic spline to approximate the discount curve, which can provide a reliable performance in several circumstances. He divides the maturity range into sub-intervals using k knot points $\kappa_1, \kappa_2, \ldots, \kappa_l$ such that $\kappa_1 = 0$ and $\kappa_1 = m_N$, with m_N the longest maturity of a bond. More specifically, the cubic spline is given as:

$$f_j(m) = \begin{cases} 0 \quad \text{if } m < k_{j-1} \\ \dfrac{(m-k_{j-1})^3}{6(k_j - k_{j-1})} \quad \text{if } k_{j-1} \le m < k_j \\ \dfrac{(k_j - k_{j-1})^2}{6} + \dfrac{(k_j - k_{j-1})(m-k_j)}{2} + \dfrac{(m-k_j)^2}{2} + \dfrac{(m-k_j)^3}{6(k_{j+1}-k_j)} \quad \text{if } k_j \le m < k_{j+1} \\ (k_{j+1} - k_{j-1})\left[\dfrac{2k_{j+1} - k_j - k_{j-1}}{6} + \dfrac{m - k_{j+1}}{2}\right] \quad \text{if } k_{j+1} \le m \end{cases} . \tag{10}$$

The resulting spline estimator is obtained through the least squares fir of the function $f(m) = \sum_{j=1}^{l} \alpha_j f_j(m)$. n being the number of observed bonds, McCulloch (1975) has suggested to place l knots, where l is the nearest integer to $\sqrt{n}$. Furthermore, these knots must be located, such that between two knots there are (nearly) the same number of bonds.

Vasicek and Fong (1982) have developed exponential splines to fit the forward rate curve. They transform the argument m of the discount function into:

$$m = -\left(\frac{1}{\alpha}\right)\ln(1-x), \; 0 \le x \le 1. \tag{11}$$

Between each pair of knot points the discount function takes the following form:

$$df_m = \sum_{i=0}^{3} \beta_i e^{-i\alpha m} \tag{12}$$

Carriere (1998) has modelled the discount function, df_m, as a survival function employed by actuaries in modelling risk default. He assumes that the knots are $\kappa_1, \kappa_2, ..., \kappa_k$ are restricted in $[0,1]$. The specification of the discount function is then given by:

$$df_m = \sum_{j=0}^{q} \phi_j [1 - \nu(m)]^j + \sum_{i=1}^{n-1} \xi_i \max(0, 1 - \nu(m) - \kappa_j)^q. \tag{13}$$

where $\{\phi_0, ..., \phi_q; \xi_1, ..., \xi_{n-1}\}$ are parameters that have to be estimated and $\nu(m)$ denotes any parametric discount function whose parameters must be estimated. Two survival function could be used for $\nu(m)$: the Weibull or the Gompertz function.

2.2.2. Nelson-Siegel Class Model

The Nelson–Siegel models are variations of the exponential polynomial models. There are two ways to characterize Nelson and Siegel class models: static form (see Nelson and Siegel (1987) and Svensson [1994, 1995)] and dynamic form [e.g. Diebold et al., (2005), Diebold and Li (2006)].

2.2.2.1. Static Representation of Nelson-Siegel Models

The main contribution of the Nelson and Siegel's (1987) model is the fact that it attempts to capture the implied forward-rate curve as:

$$f(m) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + \varepsilon_i \tag{14}$$

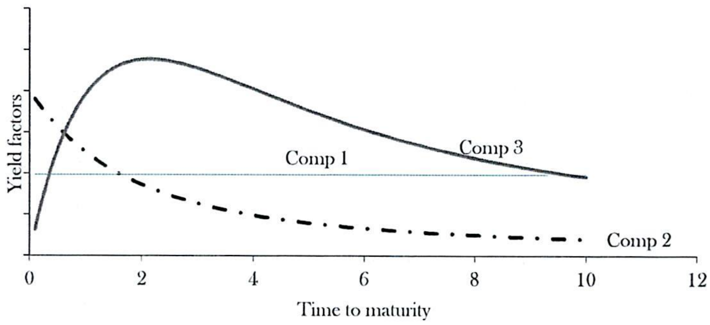
We can obtain spot rates, $s(m)$, using $s(m) = \frac{1}{T}\int_0^T f(u)du$. Therefore, the Nelson and Siegel's (1987) model to interpolate the yield curve for spot rates is written as:

$$s(m) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} \right) + \beta_2 \left(\frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} - e^{-\frac{m}{\tau_1}} \right) + \varepsilon_i \tag{15}$$

where β_0, $\tau_1 > 0, \beta_1$ and β_2 are parameters to calibrate.

The first factor is the level of the yield curve which determines the long part of the yield curve, the second one is the slope representing the short term, and the third factor is the curvature for the medium term of the yield curve [Nelson and Siegel (1987); Diebold and Li (2006)]. The different effects of factors are given in figure 13. The properties of each factor are well documented (e.g. Wahlstrøm et al. (2021) and Stander (2005)).

Figure 13 | Illustration of the factors in the Nelson-Siegel Model



Source: Author's illustration

However, some authors adopt a specification with only the level and the slope [e.g. Rudebusch and Wu (2008)] since the third component explain a smaller proportion of interest rate variance than the first two factors (see for instance De Pooter, 2007). Therefore, the model is written as:

$$s(m) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} \right) + \varepsilon_i \tag{16}$$

In the canonical Nelson-Siegel model, the slope and curvature factors display a rapid convergence to 0 [see for instance Diebold and Li (2006)] so that the level factor is the only driver of the yields curve in the long run (Diebold and Rudebusch, 2013). To lessen these limitations, several extensions have been proposed by Svensson (1995), Bjork and Christensen (1999), De Pooter (2007) and Christensen et al. (2009, 2011).

The Svensson's model (1994, 1995) incorporates a second curvature factor which allows to reflect a second hump. This increases the flexibility of the yields curve and perform better in fitting observed market data. More exactly, Svensson (1994) specifies the forward rate as follows:

$$f(m) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \left[\frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right)\right] + \beta_3 \left[\frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right)\right] + \varepsilon \tag{17}$$

where $\tau_2 > 0$ and β_3 are coefficients whose properties are well documented in many researches [see e.g. Wahlstrøm et al. (2021) and Stander (2005].

The characterization of spot rates for Svensson (1994, 1995) is thus given by the following functional form:

$$s(m) = \beta_0 + \beta_1\left(\frac{1-e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}}\right) + \beta_2\left(\frac{1-e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} - e^{-\frac{m}{\tau_1}}\right) + \beta_3\left(\frac{1-e^{-\frac{m}{\tau_2}}}{\frac{m}{\tau_2}} - e^{-\frac{m}{\tau_2}}\right) + \varepsilon \tag{18}$$

The Svensson's model leads to a better in fit of the yields curve [e.g. Diebold and Li (2006) and Gurkaynak et al. (2007)]. Gurkaynak et al. (2007) suggests that the term structure slopes down at the very long maturities, and thus the fourth factor is important to reproduce a second hump for long maturities.

The extensions of the Nelson-Siegel model introduced by Bjork and Christensen (1999) and De Pooter (2007) introduce one dimension by restricting the additional non-linear parameter to be double the value of the existing non-linear parameter in Nelson-Siegel. By contrast, the De Pooter's (2007) model consider an additional curvature term while Bjork and Christensen (1999) introduce a novel slope term. Even with these extensions, the flexibility of the Nelson-Siegel class models is constrained compared to the spline function, especially if the parameter λ is pre-set to permit linear estimation.

The exponential model also displays a similar functional form that the Nelson-Siegel class model (see Anderson et al. (1997) and Stander (2005)). The specification of the forward rate is written as:

$$f(m) = \sum_{i=0}^{n} \beta_i e^{-c_i m} + \varepsilon \tag{19}$$

where $c_i > 0$ for all $i \geq 1$ and $c_i > c_{i+1}$ for all i. Cairns (1998) has adopted a similar specification for $n = 5$, as:

$$f(m) = \beta_0 + \sum_{i=1}^{4} \beta_i e^{-c_i m} + \varepsilon\,, \tag{20}$$

to adjust the UK structure term of interest rates. By fixing the c_i as coefficients $c_i = (0,2; 0,4; 0,8; 1,6)$, only five parameters should be estimated. This lessens the risk of non-unique solutions. Cairns (1998) improves the Nelson-Siegel-Svensson models since it allows up to three inflexion points and avoids catastrophic jumps which can appear in the other two models.

2.2.2.2. Dynamic Representation of Nelson-Siegel Models

Static yield-curve models presented in the previous subsection involve fitting a mathematical function to the yield-tenor relationship. The focus of this subsection consists to present the dynamic yield-curve model describing how a given yield curve evolves over time. Initially, the Nelson–Siegel model was intended to help fit the yield curve in static way.

Diebold and Li (2006) has modified the canonical Nelson and Siegel model, including the influence of time as in equation [21] to enable a variety of time-varying yield curve shapes:

$$s(m,t)=\beta_{0,t}+\beta_{1,t}\left(\frac{1-e^{-\frac{m}{\tau_{1,t}}}}{\frac{m}{\tau_{1,t}}}\right)+\beta_{2,t}\left(\frac{1-e^{-\frac{m}{\tau_{1,t}}}}{\frac{m}{\tau_{1,t}}}-e^{-\frac{m}{\tau_{1,t}}}\right)+\varepsilon_t . \qquad (21)$$

This modification transformed a static curve-fitting technique into a dynamic version of yield-curve. $\beta_{0,t}$, $\beta_{1,t}$, and $\beta_{2,t}$ are now interpreted as latent factors [see Diebold and Li (2006) and Diebold et al. (2008)] model. For that purpose, we only fit the Nelson–Siegel model to one's yield-curve for every date over one's sample period. Diebold and Li (2006) suggest that there is little loss of generality if we assume the constancy of $\tau_{1,t}$. So the model we employ is written as:

$$s(m,t)=\beta_{0,t}+\beta_{1,t}\left(\frac{1-e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}}\right)+\beta_{2,t}\left(\frac{1-e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}}-e^{-\frac{m}{\tau_1}}\right)+\varepsilon_t \qquad (22)$$

2.3. Equilibrium Models

Equilibrium models derives yield curves from stochastic process that conducts interest rates, and deduce a characterization of zero curves in an arbitrage-free market. The model parameters are suitable with the bond prices implied in the zero coupon yield curve.

More specifically, equilibrium models are based on the dynamics of short rates, r_t, which is given as:

$$dr_t=\mu(t,r_t)dt+\sigma(t,r_t)dw_t \qquad (23)$$

with $\mu(t,r_t)$ the drift and $\sigma(t,r_t)$ the volatility of money-market rates under the historical probability. These models are arbitrage-free and assume lognormal bond prices. This produces an affine-yield solution in which bond yields are linear functions of short-term rate. Such linearity induces a simplification in the pricing of fixed-income securities and some contingent claims.

The theoretical model proposed by Vasicek (1977) describes the levels of yields using a single exogenous factor. He proposes a zero-coupon yield curve that is arbitrage-free and supported by the current level of short-term rates. The functional specification of the short-term rate is given by:

$$dr_t = \alpha(\gamma - r_t)dt + \sigma dw_t \tag{24}$$

where $\gamma > 0$ is the expected long-term level of the money-market rate, $\alpha > 0$ represents the speed of adjustment to the mean, $\sigma > 0$ is the volatility of the money-market rate. The drift component $\alpha(\gamma - r_t)$ ensures mean reversion of the interest rate towards the long-term value γ. More exactly, when r_t is higher than the long-term money-market rate, the stochastic differential equation (SDE) has a negative drift. By contrast, when r_t is low, the process has a positive drift. w_t is a wiener process which has two basic properties: (i) $dw_t = \varepsilon_t\sqrt{dt}$ with $\varepsilon_t \sim \aleph(0,1)$, and (ii) the values for dw_t are independent for two different short interval with a length dt.

Using the Itô's lemma, the solution of the SDE is given by:

$$r_t = r_s e^{-\alpha(t-s)} + \gamma\left(1 - e^{-\alpha(t-s)}\right) + \sigma\int_s^t e^{-\alpha(t-u)}dW_u \tag{25}$$

For $t\text{-}s=1$, the exact discretization is given by:

$$r_{t+1} = r_t e^{-\alpha} + \gamma\left(1 - e^{-\alpha}\right) + \sigma\sqrt{\frac{1-e^{-2\alpha}}{2\alpha}}\varepsilon_t \tag{26}$$

We can re-write [26] in an autoregressive model as follows:

$$r_{t+1} = \beta_0 + \beta_1 r_t + \beta_2 \varepsilon_t \tag{27}$$

with $\beta_0 = \gamma\left(1 - e^{\alpha}\right)$, $\beta_1 = e^{\alpha}$, and $\beta_2 = \sigma\sqrt{\frac{1-e^{-2\alpha}}{2\alpha}}$. Using an identification we obtain α, γ and σ as follows:

$$\begin{cases} \alpha = -\ln\beta_1 \\ \gamma = \dfrac{\beta_0}{1-\beta_1} \\ \sigma = \beta_2\sqrt{\dfrac{2\ln\beta_1}{\beta_1^2 - 1}} \end{cases}$$

From the process proposed for the money-market, r_t, we can derive the zero-coupon bond price by the following SDE:

$$dP_t = \mu P dt + \rho P dw_t \tag{28}$$

where $\mu = \frac{1}{P}\left(\frac{\partial P}{\partial r}\alpha(\gamma - r) + \frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\sigma^2\right)$ and $\rho = \sigma\left(\frac{\partial P}{\partial r}\frac{1}{P}\right)$. μ is the instantaneous rate of return on the bond; and ρ is the sensitivity of the bond price to changes in the money-market rate.

Although negative interest rate are observed now in advanced economies, the main problem with the Vasicek's model is that negative short-term rates can be observed.

Cox et al. (1985, hereafter CIR) have proposed an alternative one factor model for which the money-market rates are always positive or null. They have introduced the following risk-neutral process for r_t:

$$dr_t = \alpha(\gamma - r)dt + \sigma r^{\beta} dw_t \tag{29}$$

This process has the same mean-reverting drift as Vasicek's model. However, the volatility of the Brownian motion is proportional to r^{β}. The authors suggest that $\beta = 0.5$ to avoid a negative value of the interest rate for $\alpha, \gamma > 0$. A null money-market market is not possible if $2\alpha\gamma \geq \sigma^2$. More generally, when the short interest rate is close to 0, the standard deviation is very low. This, in turn, reduces the effect of the stochastic term. As a consequence, the drift factor dominates the evolution of short-term interest when the rate is close to 0.

Following the same methodology as in the Vasicek model, the exact discretization can be expressed:

$$r_{t+1} = r_t e^{-\alpha} + \alpha\gamma e^{-\alpha} + e^{-\alpha}\sigma\sqrt{r_t}\varepsilon_t \tag{30}$$

This corresponds to the following AR(1):

$$r_{t+1} = \beta_0 + \beta_1 r_t + \beta_2 \varepsilon_t \tag{31}$$

with $\beta_0 = \alpha\gamma e^{-\alpha}$, $\beta_1 = e^{-\alpha}$, and $\beta_2 = \sigma e^{-\alpha}$. As a result, we obtain the parameters of our model are given by:

$$\begin{cases} \alpha = -\ln \beta_1 \\ \gamma = -\dfrac{\beta_0}{\beta_1 \ln \beta_1} \\ \sigma = \dfrac{\beta_2}{\beta_1} \end{cases}$$

The Cox et al.'s model is a generalisation of the Vasicek formula. Indeed, when $\beta = 0$ the model corresponds to the Vasicek model, implying that negative interest rates can appear.

Ho and Lee (1986) proposes the following process:

$$dr_t = \gamma_t dt + \sigma dw_t \tag{32}$$

The drift term γ_t is time-varying. As in the Vasiseck and the CIR's models, negative rates are possible in the Ho and Lee's theoretical framework since it generates a symmetric distribution of rates in the future. Moreover, Ho and Lee (1986) do not incorporate mean reversion. For both of these reasons, models such as Hull and White (1990) and Black et al. (1990) are often preferred.

Hull and White (1990) assume that the short-term rate is normally distributed and subject to mean reversion. In the special case where $\alpha = 0$, the model is reduced to the earlier Ho and Lee model.

The SDE describing by the Hull-White model has the following functional specification:

$$dr_t = (\gamma_t - \alpha r_t)\,dt + \sigma dw_t. \tag{33}$$

When α, γ and σ are invariant, the solution of the SDE is given as:

$$r_t = r_s e^{-\alpha(t-s)} + \frac{\gamma}{\alpha}\left(1 - e^{-\alpha(t-s)}\right) + \sigma \int_s^t e^{-\alpha(t-u)} dW_u \tag{34}$$

We can also use the exact discretization to estimate these parameters.

The Black et al.'s (1990) model also known as the Black-Derman-Toy model is the first model to combine the mean-reversion of the short rate with the log-normal distribution as in equation [35].

$$d \ln r_t = \gamma_t dt + \sigma_t dw_t \tag{35}$$

Several other specifications of equilibrium models have been developed to prevent negative short rates [see for instance Duffie and Kan (1995)].

An important limit of equilibrium models with one factor is their inability to reconcile the time-series dynamics of interest rates with the cross sectional shapes of the term structure.

In this chapter, we have the main theoretical background of the yield curve. First, we have make an overview of the main traditional economic theories that explain different shapes that a term structure can display. We have seen that term structure theories can be gathered in four categories: pure expectations hypothesis, liquidity preference hypothesis, preferred habitat hypothesis and market segmentation theories. The first three theories are based on the existence of a close link between interest rates with different maturities, while the market segmentation theory postulates that there exists no relationship between short, medium and long interest rates.

Second, we have make a review of main models used to derive yield curves. The literature proposes regression-type yield curve models, empirical models which are mainly driven by parsimonious approaches proposed by Nelson and Siegel, and equilibrium models which make assumptions about economic variables by taking phenomena such as mean reversion and volatility roll-down into account. However, all these models are not able to capture at the same time the form of the yield curve at a given point in time and the dynamics of the yield curve through time. As a result, the extracted yield curves always present some shortcomings.

In the next chapter, we construct yield curves among CEMAC countries. The extraction yield curve allow us to test the evidence of different term structure theories exposed in this chapter. Moreover, we test the robustness of several empirical approaches on fixed income markets data of the CEMAC zone.

Chapter III | Modelling Yield Curves of CEMAC Countries

The term structure is at the center of fixed income markets. Portfolio managers use the yield curve to assess the relative market fair value of investments across the maturity spectrum. The yield curve indicates the returns that are available at different maturity points and is therefore very important to fixed income fund managers, who can use it to assess which point of the curve offers the best return relative to other points.

In the Economic Community of Central African States (CEMAC), fixed income markets are nascent. They have registered an impressive development after the global financial crisis but they are illiquid, several member states have not extended their bond maturities, the bid-ask spread is important, and issue sizes and secondary market activity are not deepen compared to the West African Economic and Monetary Union (WAEMU) which presents the same characteristics. More specifically, most Government issues do not exceed 5 year maturity, except in Cameroon and Gabon. Equatorial Guinea which presents good economic outlook has not yet issued a maturity greater than 1 year and coupon rates proposed by the country for T-Bills often exceed 7 percent. Moreover, many transactions backed on government securities lead to important discounts on the secondary market. Therefore, a consistent approach in the construction of the yield curve could help to increase liquidity and efficiency. More exactly, the construction by extrapolation of a segment of the long-dated maturities yield can help to create a certain appetite for these securities.

The main goal of this chapter is to estimate and fit yield curves for all CEMAC countries. To reach that goal, we employ daily data for fixed income instruments issued or traded for the period covering August 01, 2019 to January 01, 2021. Data consist on transactions on repo reverse, T-Bills, bond auction and bond syndication. We extract yield curves using the canonical Nelson-Siegel model and the extended version of Svensson.

In the Sub-African context, the empirical literature on zero-coupon yield curve is rare although some studies exist in West and Southern Africa [see for instance Gbongué and Planchet (2015), Gbongué et al. (2017), Lartey and Li (2018), Lartey et al. (2019), Gbongué (2019a, 2019b)]. Moungala (2013) uses the ACP method to construct the yield curves for South Africa, France and the United States. Gbongué and Planchet (2015) propose a complete methodology for constructing a zero-coupon rate curve, which integrates the key rate, the long-term real economic growth rate and targeted inflation. Gbongué and Al. (2017) propose an economic scenario generator (ESG) adapted to the CIPRES zone based on nominal interest rates and the harmonized index of consumer prices. Lartey and Li (2018) conclude that the thermite cubic spline interpolation model fits the Ghana data better than models. In our best knowledge, this is the first empirical study within the context of the CEMAC zone.

The chapter relies on four sections structured as follows. Section 1 presents the methodological framework. In section 2, we make preliminary analyses and the last section presents the empirical results and some discussions.

1. Methodology for Modeling Yield Curves

The present section presents the methodology used to build a yield curve at a given time. The methodology relies on four stairs. First, we present the approach used for measuring yields of fixed income securities. Second, we present the estimation method. More exactly, we describe the pricing function of bonds and the optimization routine for deriving parameters estimates. Third, we depict some computational aspects for implementation: initialization procedure for the optimization algorithm and constraints. Finally, we present the different empirical models tested.

1.1. Yield Measurement

In this sub-section, we present the methodology that we employ to obtain yield to the maturity (YTM) of a fixed income instrument. In this study, three instruments are used to derive yield curves: T-Bills, T-Bonds, and repos. Bonds syndication were not included since transaction costs on the regional stock markets are expensive and induce abnormal high yields. We also exclude OTZ in our sample.

The first step for building government yields curves is a data harmonization, i.e., converting T-Bills rates into an actuarial actual/365 basis. The day-count convention for the T-Bills is Actual/360 while it is actual/actual[2] for T-Bonds. These different conventions adopted in money markets compared to bond markets induce some inconvenient when comparing yields [see for instance Choudhry (2014, 2019)]. For that purpose, we must adjust a money market-equivalent yield for bond instruments to make reasonable comparisons. The money market, r_m, of T-bills is expressed as

$$r_m = \frac{360(N-P)}{nP},$$

with N the facial value, P the dirty price, and n the tenor. The adjustment of the money market yield is given by equation 36, which shows the adjustment required to the Actual/Actual day-count convention:

$$r_a = \left(1 + \frac{n}{360} r_m\right)^{\frac{365}{n}} - 1 \qquad (36)$$

Based on auction results carried out during the week, we report the computed yield on the weekly yield curve with the associated tenor. When there is no new issues for a specific tenor, the last effective yield is retained.

[2] An exhaustive list of the conventions is given in the methodological note proposed by the central bank of central states in 2018.

The government repurchase market or repo reverse market is used as the alternative instrument to T-bills for adjusting the short-term curve accurately. This choice is justified by the fact that BTA instruments are restricted to licensed primary dealers, and the yield of these instruments reflects its liquidity and risk-freeness [see Duffie (1996)].

The gross redemption yield (GRY) or YTM, r_a, is given by internal rate of return (IIR) on the security, the rate that equates the value of the discounted cash flows to its current dirty price [see Choudhry (2019)]. For T-Bonds paying annual coupons with bullet repayment, the YTM is calculated by solving the following equation:

$$P_i = \sum_{t=1}^{n_i} \frac{c_t}{(1+r_a)^i} + \frac{N}{(1+r_a)^{n_i}} \quad (37)$$

where P_i is the dirty price of bond i, N is the par or redemption payment, n_i is the number of interest periods, and c_i the coupon. The number of interest periods is the tenor.

1.2. Estimation Method

Estimation methods used in the past employ weighted least squares [see for instance McCulloch (1975)], maximum likelihood [see Litzenberger and Rolfo (1984)], linear programming [e.g. Ronn (1987)], and iterative extraction [Fama and Bliss (1987)]. In this work, we use error weighting functions as in Bliss (1997), and Gbongué (2019a, 2019b).

To set up the optimization routine, we must specify an appropriate functional form for spot rates and estimate the parameters, with the constraint that when each of the bond price is computed with this mathematical function, it will equal the observed price. The bond prices, P_i, are computed by equation (38)

$$P_i = \sum_{j=1}^{n_i} C_{i,j} df_{m_{i,j}}, \quad (38)$$

where n_i is the number of outstanding cash flows of bond i, df_m is the discount function for a maturity m, $m_{i,j}$ is the maturity from the value date to the j-th future cash flow, $C_{i,j}$ of bond i. In practice, we introduce an error component to obtain the pricing function,

$$P_i = \sum_{j=1}^{n_i} C_{i,j} df_{m_{i,j}} + \varepsilon_i .$$

The parameters of this function are estimated by an iterative technique where we minimize the weighted squared differences (WSE) between observed and fitted bond prices[3]. The WSE at a given point in time, t, is written as:

[3] The choice between yield or price error minimization is not definite and depends on the intended use of the yield curve [See Wahlstrøm et al. (2021)]. It is more appropriate to minimize yield errors when the

$$WSE_t = \sum_{i=1}^{n} w_i \left[P_{i,t}^{obs}(m) - \hat{P}_{i,t}(m) \right]^2 \quad (39)$$

where w_i is the weight of bond i, $\hat{P}_{i,t}(m)$ denotes the fitted yield and n the number of bonds used to derive the term structure. Following Bliss (1997), the weights, w_i, given below are defined by the Macauley duration, d_i, measured since there is a combination of fitted-price errors across maturities in a single week i:

$$w_i = \frac{1/d_i}{\sum_{j=1}^{N} d_j}.$$

1.3. Computational Aspects

The parameters of the empirical yield curve models are estimated with iterative methods [see Stander (2005)]. These techniques require initial values for the parameters. If good starting values are selected, the iterative procedure converges to results much faster, and may also shows a global optimum value instead of wrongly converging to a local optimum. It is possible to consider various different combinations of parameters, then choose that combination where the resulting curve has the correct shape. Within Nelson-Siegel class models, this can be a fastidious work the first time a specific yield curve function is fitted. However, once the parameters are adequately calibrated, these parameter estimates are used as the starting values for the next day, week or month, and so on. Another possible way to calibrate initial values consists to specify the range in which the parameters can vary.

For fitting our yield curves, we then consider the initial values and restrictions listed in Table 3.

Table 3 | Initial values derived from observed yields

Parameter	Initial value	Restrictions
β_0	see equation 40	$-\beta_0 \leq 0$
β_1	see equation 41	$-\beta_0 - \beta_1 \leq 0$
β_2	0	/
β_3	0	/
τ_1	1	$0 \leq \tau_1 \leq 30$,
τ_2	1	$0 \leq \tau_2 \leq 30$

Source: Manousopoulos and Michalopoulos (2009).

---

purpose is deriving interest rates for monetary policy decisions. In contrast, minimizing price errors appears more suitable if the purpose is pricing of bonds.

The initial values are derived from observed yields and consistent with the financial interpretation of the parameters as in Manousopoulos and Michalopoulos (2009). The initial values of the magnitudes of the long-term (level) factor β_0 and the short-term (slope) factor β_1 are initialized by:

$$\beta_0 = \frac{1}{3}\sum_{j=1}^{3} r_j \tag{40}$$

and

$$\beta_1 = r_s - \frac{1}{3}\sum_{j=1}^{3} r_j \ , \tag{41}$$

where r_i's are the observed yield to maturity in percent of the three securities with the longest time to maturity and r_s is the observed yield to maturity in percent of the instrument with shortest tenor. $\beta_2 = \beta_3 = 0$, and $\tau_1 = \tau_2 = 1$.

We have selected the Broyden–Fletcher–Goldfarb–Shanno (BFGS) variable metric algorithm for the yield curve estimation problem. It is an efficient and popular general purpose non-linear optimization algorithm available under the Excel Solver Toolbox.

For a computational implementation, outliers that can significantly disturb yield curve fitting or dampen the economic interpretation of parameters were removed during the preliminary visualization of data. Outliers were mainly observed for intra-group operations on the secondary market. Yields that usually result from those operations do not necessarily reflect the market and can be ignored for building a fair term structure. Technically, yields that overpass the coupon rate by more than 500 bps are not considered in our study.

Furthermore, when a maturity does not correspond to an issue, we constraint the scope to be equal to the initial value to avoid a collapse or an uncontrolled dynamics of the yield curve on the long run. This approach is suitable with the fixed income market in the CEMAC which does not provide long maturities. Many new maturities are also traded looking to previous yields offered to investors.

1.4. Models Tested

Three methods taken from the existing literature are used to estimate the yield curve: the canonical Nelson-Siegel and the extended versions of Svensson (1994).

The traditional fitting technique introduce by Nelson and Siegel (1987) has since been applied and modified by several authors, which is why they are sometimes described as Nelson and Siegel Models.

The canonical Nelson and Siegel's (1987) model suggests that the implied forward rate yield may be modelled using the following function form:

$$f(m) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) \qquad (42)$$

where $\theta = \{\beta_0, \beta_1, \beta_2, \tau_1\}$ is the vector of parameters describing the yield curve. Their properties are well documented in the literature [see for instance Stander (2005), Bolder (2015), and Choudhry (2019)]. m refers to the maturity at which the forward rate is computed. We can obtain spot rates using the following expression,

$$s(m) = \frac{1}{T}\int_0^T f(u)\,du\,.$$

Therefore, the Nelson and Siegel's (1987) model to interpolate the yield curve for spot rates is written as:

$$s(m) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} \right) + \beta_2 \left(\frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} - e^{-\frac{m}{\tau_1}} \right) \qquad (43)$$

An extended version of the Nelson and Siegel model was proposed by the Svensson (1994). This new approach allows for the humped characteristic of the yield curve. The functional form of forward rate is given by:

$$f(m) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \left[\frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right)\right] + \beta_3 \left[\frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right)\right], \qquad (44)$$

where $\tau_2 > 0$ and β_3 are additional coefficients whose properties are well documented in many researches [see e.g. Stander (2005), Choudhry (2019), and Wahlstrøm et al. (2021)].

The existing literature on the fitting of yield curve employs a graphical representation [see for instance Stander (2005)] or statistical indicators [e.g. Bolder and Streliski (1999), Anderson and Sleath (2001), Brousseau (2002), Aljinovic et al. (2012), and Kovachev and Simeonov (2014)]. In this study, we only employ graphical approach to examine the quality of different tested models.

2. Data and Preliminary Analyses

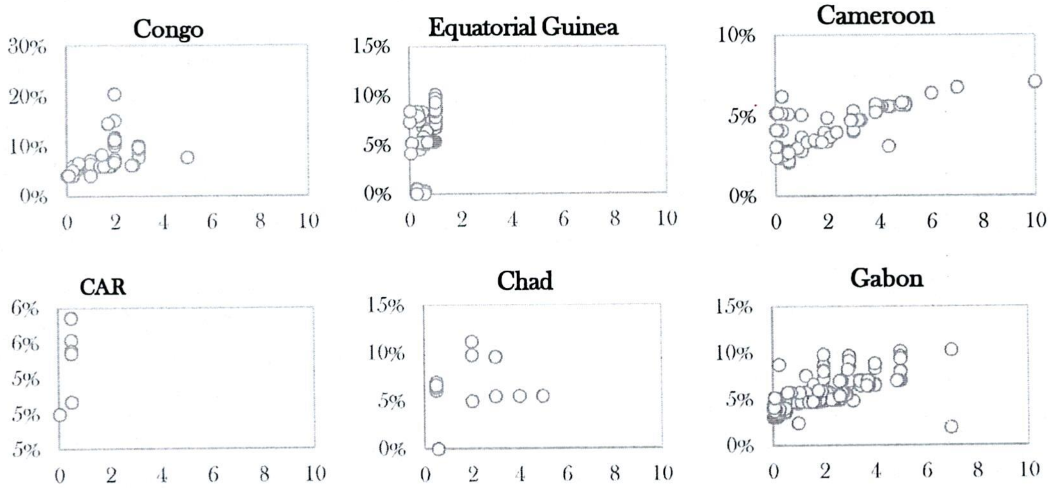
In this section, we present data source and summary statistics related to yields on fixed income markets. We also make a descriptive analysis of the monthly regulatory yield curve.

2.1. Data Source and Summary Statistics

The data consist of weekly data for fixed income instruments issued or traded for the period covering August 01, 2019 to January 01, 2021. For risk-free yield curve, instruments include money assets (repo reverse on negotiable government debt securities and T-Bills) and bonds auction on the regional securities market[4]. We extract data from both from both the primary and the secondary market.

Figure 14 outlines a collection of CEMAC countries bond yields from August 2019 to December 2020.

Figure 14| Bond Yields in CEMAC Countries



Source: Author's Calculations
Notes: This is the collection of Treasury bond yields from August 2019 to December 2020.

Yields for more than 10 year maturities have not been already observed yields in All CEMAC countries. Equatorial Guinea has not resorted to an issue greater than one year during the period under consideration. Cameroon, Gabon and Congo display the high number of observed bond yields on the primary and the secondary market. Except for Cameroon and Gabon to a lesser extent, it is difficult to conclude to an evident relationship

[4] By contrast to the BEAC methodology, we exclude bond syndication on the regional financial market since they lead to yields outliers which can significantly impact the fitting and the economic interpretation of the yield curve.

between individual yields and their associated tenors. The reason is simple, the market is not deep and illiquid Treasury securities has an extremely irregular weekly and monthly issuance pattern in the zone. Short dated securities with tenor lower than one year range from 2 to 8 percent in Cameroon. We can also see that yields asymtotically converge to 8 percent. In contrast, Gabon short dated maturities tend to provide lower yields than Cameroon but investors require higher yields for long dated securities. Congo offer higher yields for both short and long dated maturities than Cameroon and Gabon.

Table 4 gives us a prefiguration of the yield curve for each country in the CEMAC zone for the period ranging from January 2020 and December 2020. We report summary statistics for bond yields at representative maturities.

Table 4 | Summary Statistics of Yields by Tenor

	Mean	Std dev.	Min	Max
Cameroon				
0-1	4.40%	1.10%	2.11%	6.22%
1-2	4.30%	0.90%	3.39%	5.39%
2-3	4.46%	0.64%	3.51%	5.59%
3-4	5.30%	0.51%	4.13%	5.82%
4-5	5.41%	0.81%	3.13%	5.84%
5-6	5.75%	0.06%	5.70%	5.82%
6-7	6.41%	-	6.41%	6.41%
7-8	6.76%	-	6.76%	6.76%
9-10	7.08%	0.01%	7.07%	7.09%
Congo				
0-1	5.45%	1.10%	4.06%	7.13%
1-2	6.38%	0.78%	5.97%	8.32%
2-3	10.31%	1.81%	6.18%	11.75%
3-4	9.36%	0.83%	7.74%	10.10%
5-6	7.72%	-	7.72%	7.72%
Gabon				
0-1	4.50%	0.76%	2.44%	8.75%
1-2	5.14%	0.63%	4.73%	7.54%
2-3	6.46%	1.35%	4.98%	9.82%
3-4	7.30%	1.11%	4.84%	9.68%
4-5	7.17%	0.86%	6.23%	8.85%
5-6	9.29%	0.93%	8.00%	10.18%
6-7	1.25%	1.00%	0.55%	1.96%
7-8	10.32%	-	10.32%	10.32%
Equatorial Guinea				
0-1	6.19%	2.33%	0.24%	10.11%
CAR				
0-1	5.52%	0.21%	5.27%	5.74%
Chad				
0-1	6.62%	0.22%	6.10%	6.93%
2-3	8.65%	3.24%	5.00%	11.18%
3-4	7.54%	2.88%	5.50%	9.57%
4-5	5.50%	-	5.50%	5.50%
5-6	5.50%	-	5.50%	5.50%

Source: CRCT/Author's Calculations.

On the whole, yield curves are upwardsloping. Except in Cameroon, yield volatility tends to increase with maturity since long dated securities issues are rare on the regional government securities market.

Cameroon registers the highest uncertainty for short dated maturities where yields can vary up to 1.10 percent. This can be explained by the fact that 13, 26 and 52 week T-Bills usually provide low yields to investors (lesser than 4 percent) whereas repo reverve transactions and deals on the secondary market are often achieved at 5 percent at least. The difference is also visible by the range of values, the minimum yield registered is 2.11 percent and the maximin is 6.22 percent. Apart from money market yield, the term structure has a normal shape for medium and long dated maturities.

Gabon presents the lowest maturity for short-term securities. This can be explained by the fact that repo reverse transactions usually deliver similar yields than T-Bills. However, some deals among primary dealers or intra-group transactions can generate important yields for maturities lower than 2 years. That is why we can register a yield equal to 8.75 percent for maturites lower or equal to 1 year. The highest uncertainty is recorded for maturities comprise between 2 and 3 years.

Congo registers higher yields than in Gabon and Cameroon over the yield curve. Treasury securities also records the highest uncertainty for maturities comprise between 2 and 3 years. Yields range from 6.18 to 11.75 percent.

Equatorial Guinea and Chad have provided more than 6 percent on average to investors for maturities lower than 1 year. In Chad, this high yiedl is determined by poor economic fundamentals and moderate oil price. In contrast, Equatorial is perceived as risky for invesors because of its legal framework and its low integration with other CEMAC coubtries.

Regarding Central African Republic, we can see that issues remain lower than 1 year and yields served by the Treasury are greater than that of Congo. This is not surprising looking to its macroeconomic foundamentals and the political instability in the country.

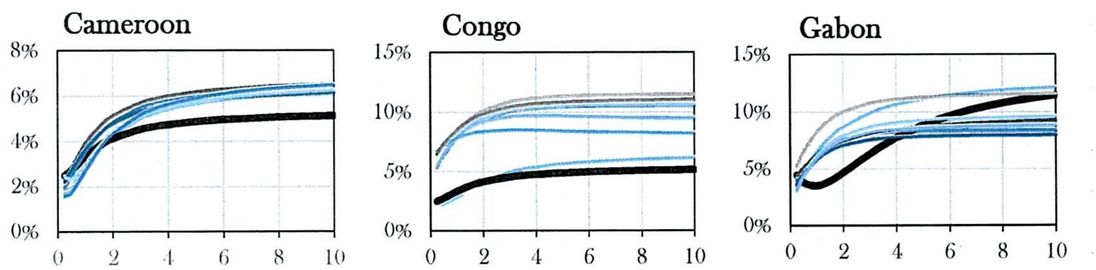
2.2. Examining the Regulatory Yield Curve

Figure 16 depicts a simply graphing of all available Treasury yield curves together against their tenor published by the central bank of central African states between January 2020 and December 2020. We can notice that yields may take a wide range of values across all tenors. Short-term rates, for instance, range from 2 to slightly less than 4 percent over this time interval in Cameroon while they range from 4 to 8 percent in Congo and Gabon.

Long dated maturities cover the spectrum from 4 to 7 percent in Cameroon while they can reach 12 percent in Congo and Gabon. The yield curve can also take a variety of different shapes. It may be upward sloping, flat, or downward sloping. Sometimes the curve moves upwards almost linearly (e.g. Gabon), while in other cases it slopes up steeply and then flattens off (Congo).

It is worth mentioning that the central bank has not yet published yield curves for Equatorial Guinea and Central Africa. Looking to Chad, only the yield curve for February 2017 is available.

Figure 15 | Central Bank Yield Curve January 2020 - December 2020

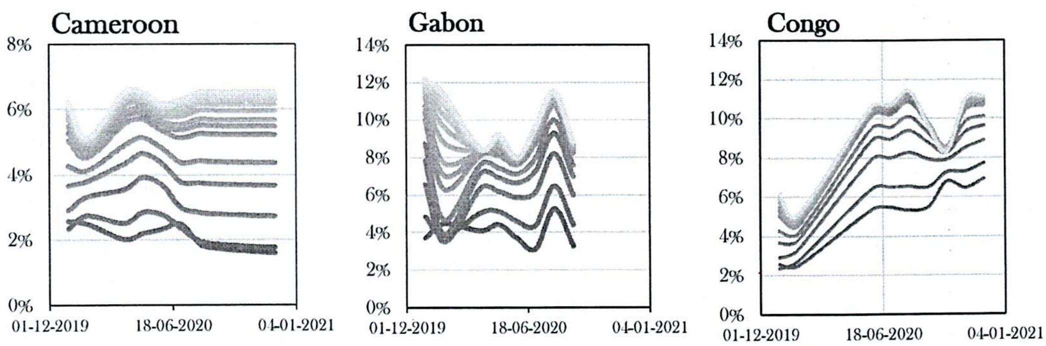


Source: BEAC

Figure 16 shows the evolution of monthly spot rates for fixed maturities of 3 months, 6 months, 1, 1.5, 2, 3, 4, 5, 6, 7, 8, 9, and 10 years.

The figure reveals that yields for long dated maturities have significantly collapsed in February 2020 while yields for medium term securities have registered an upward. It is also important noticing that yields for short dated maturities have registered a structural decrease during the year 2020 in Cameroon although the central bank has increased the interest rate for tenders. Looking to the regulatory yield curve, yields required by investors have collapsed in Gabon in February 2020. By contrast to Cameroon, yields have also significantly increased after the restrictive monetary policy taken by the central bank in September 2020. Not surprisingly, yields have recorded an upward dynamics in Congo.

Figure 16 | Evolution of the Yield Curve



Source: Author's Calculations

Note: The lines have unique colors from blue shades for the shortest maturities to white for the longest maturities.

3. Empirical Results

The empirical results begin with a static analysis of our empirical results. We construct a yield curve for the entire period covering August 01, 2019 to December 2020. Thereafter, we make a dynamic analysis by constructing weekly yield curves. We conduct a weekly analysis to match with the weekly valuation of mutual funds. More specifically, an extraction of weekly curves could be useful for Asset Management Companies (AMC) to measure the net asset value per share of their different funds.

3.1. Static Yield-Curve Fitting

The empirical estimates of our different models for the entire period under consideration are given in table 5. The level factor is greater than 3.5 percent for all CEMAC countries. This result is consistent with Interest Rate on Tenders (TIAO) offered by the central bank during the period under consideration.

Table 5 | Calibrated Models

		Chad		Congo		CAR		E.G.		Gabon		Cameroon	
Models		Opt.	Init.	Opt.	Init.	Opt.	Init.	Opt.	Init.	Opt.	Init.	Opt.	Init
Nelson and Siegel 1987	β_0	0.11	0.11	0.12	0.12	0.12	0.06	0.10	0.10	0.08	0.07	0.10	0.10
	β_1	-0.06	-0.06	-0.08	-0.08	-0.02	-0.01	-0.06	-0.06	-0.01	-0.05	-0.07	-0.08
	β_2	0.00	0.00	0.00	0.00	-0.18	0.00	0.00	0.00	-0.15	0.00	0.00	0.00
	λ_1	1.00	1.00	1.00	1.00	0.24	1.00	1.00	1.00	0.69	1.00	1.91	1.00
Svenson 1994	β_0	0.11	0.11	0.12	0.12	0.12	0.06	0.10	0.10	0.08	0.07	0.10	0.10
	β_1	-0.06	-0.06	-0.08	-0.08	-0.04	-0.01	-0.06	-0.06	-0.01	-0.05	-0.07	-0.08
	β_2	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	-0.06	0.00	0.31	0.00
	β_3	0.00	0.00	0.00	0.00	-0.19	0.00	0.00	0.00	-0.09	0.00	-0.29	0.00
	λ_1	1.00	1.00	1.00	1.00	0.91	1.00	1.00	1.00	0.68	1.00	3.35	1.00
	λ_2	1.00	1.00	1.00	1.00	0.37	1.00	1.00	1.00	0.69	1.00	4.34	1.00

Source: Author's Calculations

Notes: Opt. refers to the optimal value after the calibration and Init. Refers to the initial values before the calibration.

These estimated parameters allow us graphing three building blocks of yield curves for CEMAC countries including spot, forward and discount curve.

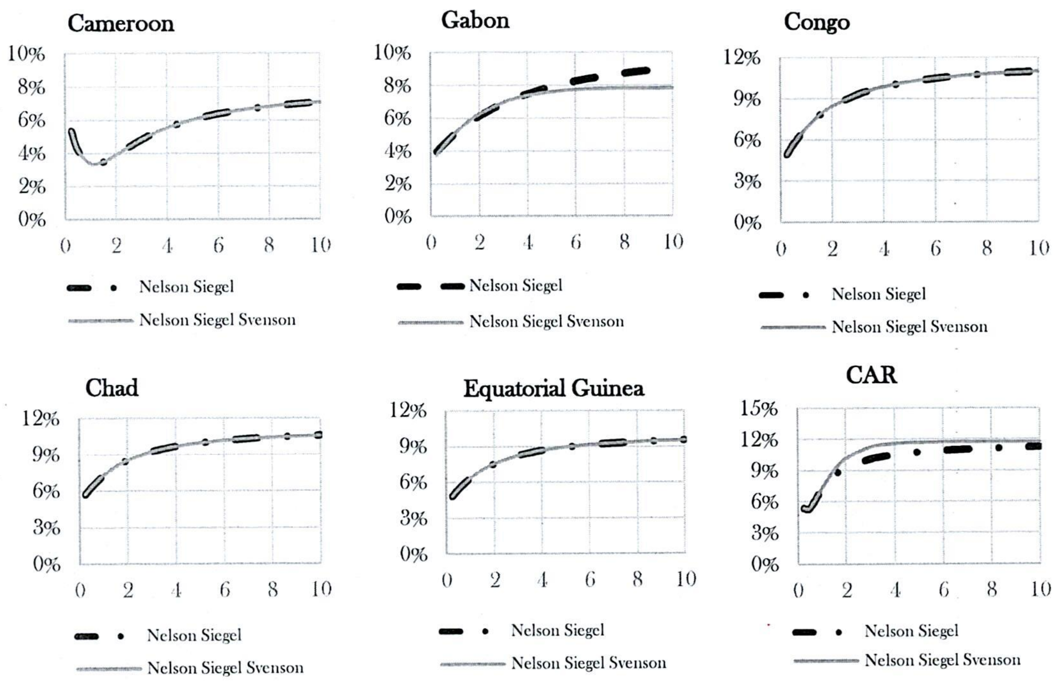
Looking to spot curves in Figure 17, we notice that risk-free rate curves obtained are increasing, with positive slopes. This is consistent with summary statistics and economic outlook in the CEMAC zone. However, some significant heterogeneities appear among CEMAC countries.

We can see that the Cameroon yield curve is consistent with the market segmentation theory. Each maturity has a specific shape. Repo reverse government securities operations lead to an increase of money market rates. After that, the yield curve displays the expected positive sloping across maturities.

Repo reverse operations do not significantly impact the shape of the yield curve in Gabon. The term structure has a normal profile but differences exist among models. The Nelson-Siegel model induces significant higher yields than in Cameroon for long dated securities while the Svensson version does not generate such important differences.

To avoid an uncontrolled dynamics, we formulate some views to fit the Congo yield curve. We assume that the medium factor is comprised between 6 and 7 percent, which is the observed yield for 2 to 4 years maturities. We also constraint the slope factor under 12 percent which can be considered as the ultimate forward rate[5] (UFR). Empirical results show that the speed of convergence is greater than 10 years. Results also show that that the canonical Nelson-Siegel model and the extended version of Svensson leads to the same empirical yield curve for the entire period. Short and long dated maturities offer higher yields to investors than in Gabon and Cameroon.

Figure 17 | CEMAC countries Yield Curve (August 2019-December 2020)



Source: Author's Calculations.

All Nelson-Siegel class models lead to the same term structure for Chad government securities. The derived yield curve corresponds to the initial parameters. This result is suitable with the low level of transactions and irregular issuance by the national Treasury. However, the results are intuitive since Chad prefers an external debt to the regional

[5] This interest rate is set after some discussions with professionals.

securities market. The country does not have an important external debt. In 2018, the existing commercial debt was restructuring. Moreover, the rise of oil price after the collapse registered in 2018 has allowed the country to improve its public finance.

Using traditional calibration, the extracted yield curve in Equatorial Guinea from the Nelson-Siegel and the Svensson models overpasses 30 percent for 10 year dated maturities[6]. Although this result is in line with high yields observed for short dated maturities, government securities cannot offer more than 12 percent coupon rate since Equatorial Guinea does not have a significant external and internal debt even if oil prices are moderate on international markets. Therefore, we fit the long part of the yield curve considering that Equatorial Guinea displays better long term fundamental than Congo. Results show that long dated maturities range from 7 to 9.5 percent, which is lower than in Congo.

Looking to Central African Republic, we employ the liquidity preference theory to calibrate the medium and the long part of the yield curve. We constraint the curve to increase and we make the hypothesis that the long term factor be greater or equal to that of Congo. This is in line with the economic fundamentals. Although Congo displays a high level of risk for investors, its economic outlook are better than in the Central African Republic. Figure 17 shows that the calibration leads to a coherent yield curve. Short along dated maturities offer greater yields than those obtained in Congo.

From Figure 18, we can conclude that the Nelson-Siegel and Nelson-Siegel-Svensson display similar findings.

Looking to Cameroon, we notice that all models present significant errors for short dated maturities. This can be explained by the fact that both T-Bills and repo reverse to fit the short part of the yield curve. In Cameroon, T-Bills offer low yields to investors while repo reverse operations backed on government securities propose higher yields. This therefore results to low fitting quality for short dated maturities.

The extended version of Svensson provides the best fitting of the yield curve in Gabon. Its flexibility allows a better adjustment of long dated maturities than the canonical Nelson-Siegel model. Moreover, repo and repo reverse operations deliver similar yields than T-Bills. This is the reason why fitted-price errors are lower than in Cameroon for short dated maturities.

Fitted-price errors are relatively stable in Congo. They are most important for 3 years maturities. This can explained by the fact that medium and long fated securities of Congo are not liquid. Even when they are traded on the secondary market, they record import discounts. Therefore, the fitting-price of these securities is associated to a high uncertainty.

Fitting-price errors increase with maturity in Chad. This is not surprising and corresponds to the fact that Chad does not usually resort to medium and long dated maturities. Both models conduct to the similar quality of fitting.

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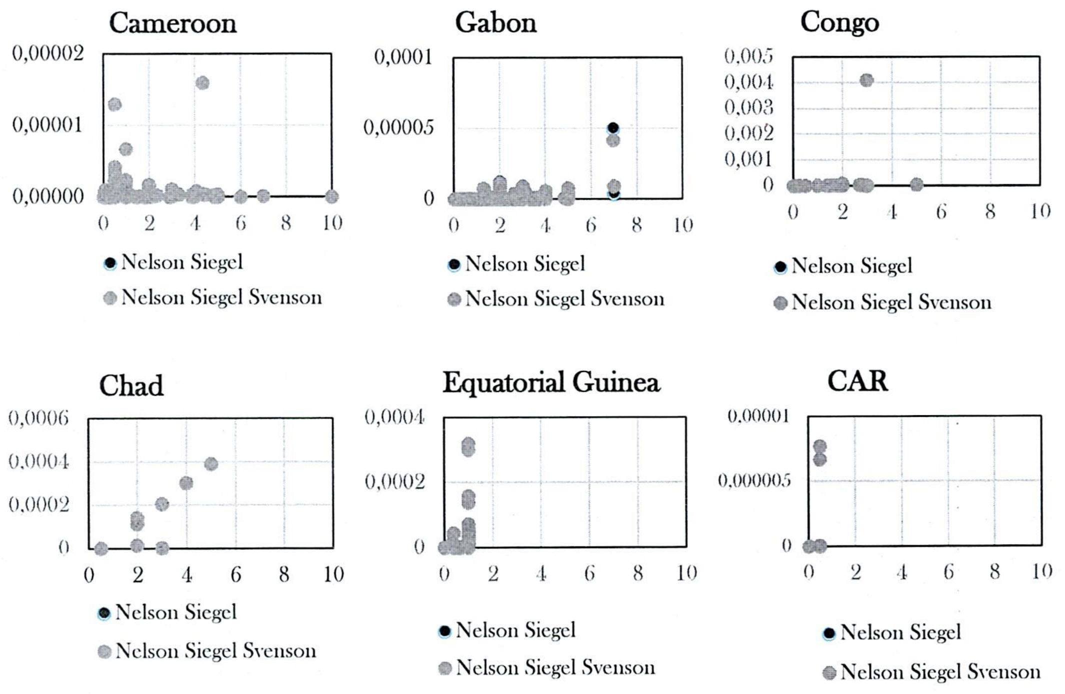
[6] These results are available upon request.

Fitting-price errors cannot be useful in Equatorial Guinea and Central African Republic because long dated government securities do not exist. However, we can notice that errors are most important for the highest maturities. This is mainly explained by expectations uncertainty that we have adopted to fit these yield curves.

Figure 18 | Fitted-Price Squared Errors



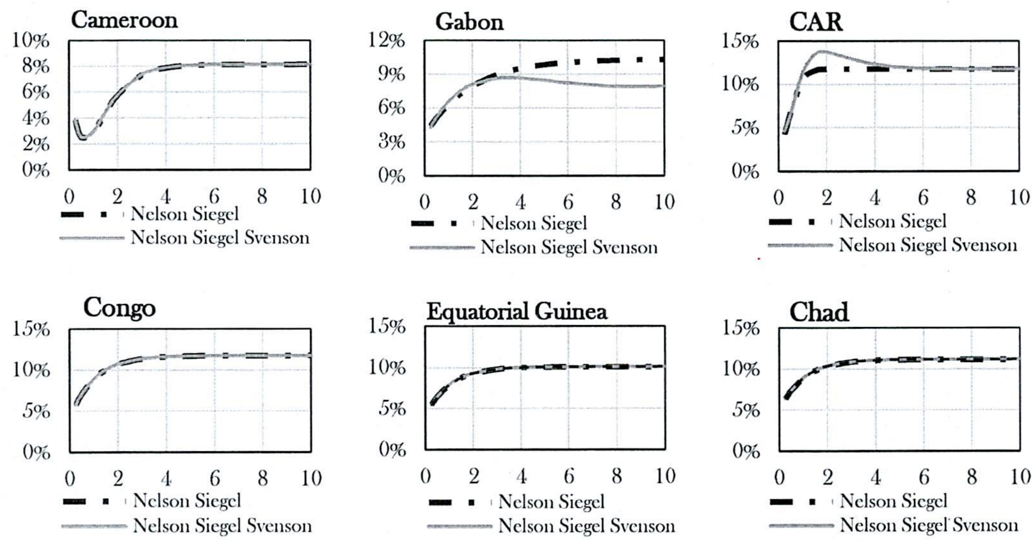
Source: Author's Calculations.

The implied forward rate curves are given in figure 19. We can see that the Nelson-Siegel and the extended version of Svensson lead to the same results in most cases (Cameroon, Congo, Equatorial Guinea and Central African Republic). The results are also intuitive and suitable with the results obtained for spot curves.

Forward rates are higher than spot rates in Chad, Gabon, Equatorial Guinea, and Congo. This is in line with the fact that their spot curve has an upward sloping spot curve, justifying the fact that forward curve is above spot curve spot curve. The gap is more pronounced for the canonical Nelson-Siegel model in Gabon since long dated maturities offer higher yields than the Svensson model.

By contrast, we notice that short forward rates are lower than spot rates while medium and long dated maturities have higher forward rates than spot rates in Cameroon and Central African Republic. This is theoretically explained by the fact that if the spot curve is downward sloping, then the forward curve is below the spot curve.

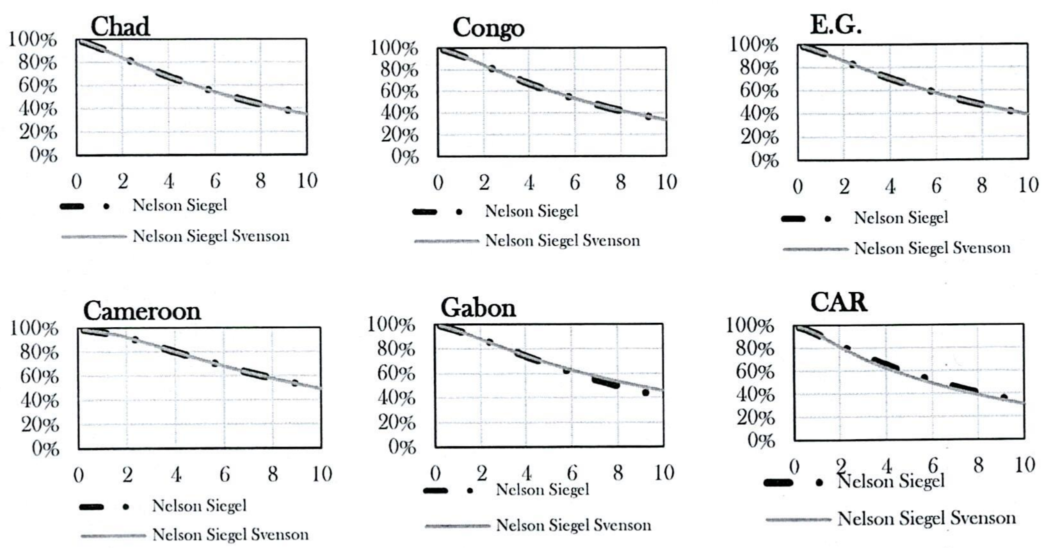
Figure 19 | Forward Yield Curve (August 2019-December 2020)



Source : Author's Calculations.

The discount functions block associated to the estimated parameters is given in figure 20 below. All discount functions display good properties by decreasing with maturities. The discount factor collapse to around 35 percent for 10 year maturities in all CEMAC countries.

Figure 20 | Discount Function Curves



Source: Author's Calculations.

3.2. Dynamic Analysis of Yield Curve

In the previous paragraph, we have focused our analysis on the fitting of the yield curve for the period ranging from August 2019 to December 2020. In this sub-section we make a dynamic analysis of the yield curve. The analysis has two stairs. First, we examine yield curve displacement across time. Second, we analyze the evolution of yield for selected maturities across time.

Yield curves for the period covering January to December 2020 are given in figure 21. We can see how the yield curve moves among CEMAC countries.

The yield curve dynamics in Cameroon is coherent with the entire period dynamics and stable across time. The fitting of the money segment by repo rate induces that short-dated securities deliver higher yields that securities whose maturities range from 1 to 2 years. This is consistent with the market-segmentation theory. Banks highly resort to repo reverse operations backed to Cameroon government securities so that the short the cost of money increase. 1-2 year securities are not highly traded and operations on the secondary do not result to important discounts so that the yield is near to coupon rate. The long part of the Cameroon yield curve is necessary and required by insurance companies located in the CEMAC zone.

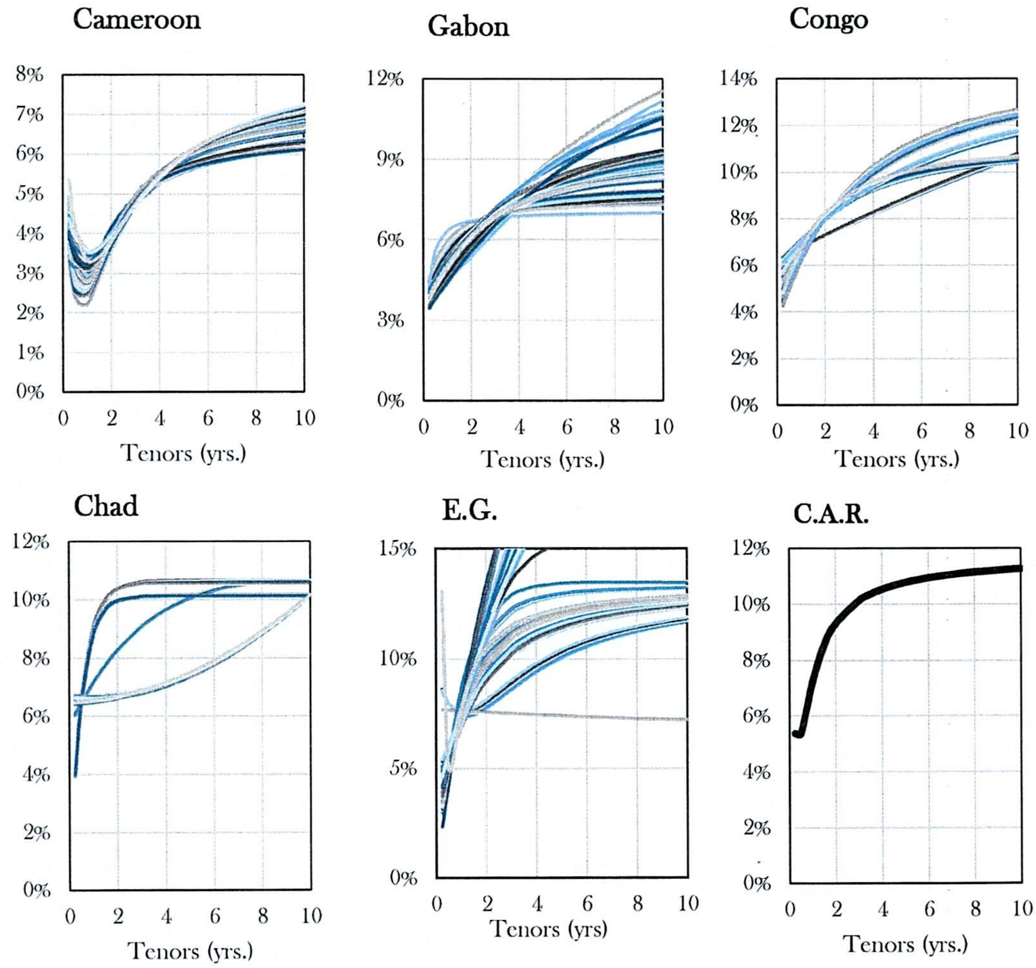
The shape of the yield curve significantly varies in Gabon. The short remains relatively stable across time while the long part tends follows a vertical displacement. More specifically, yield of short dated securities (lower than 1 year) ranges from 4 to 6 percent. By contrast, the range is more pronounced for long dated maturities. Yields range from 7 to 11 percent. This suggest that there is an important interest rate risk for bond portfolio which has an important exposure to long dated maturities in Gabon.

The shape of the yield curve does not significantly change in Congo except in January 31, 2020 where the trajectory exploded. This period corresponds with the revelation of hidden debts. Therefore, Country has registered important costs issues and transactions on their public securities have offered important discounts to investors. However, the interest risk for bond portfolio is not pronounced as in Gabon.

The yield curve of Equatorial Guinea also displays important displacements. Contrarily to Gabon which present a relative stability on the short area, all the segments of the yield curve register significant moves. This result is consistent with the last liquid observed point (LLP) which is equal to 1 year. Therefore, the extracted yield curves are associated to high uncertainty.

The yield curve of Chad maintains a relative constant shape because the country does not often resort to the regional public securities market for its financing needs. The yield curve of Central African Republic does not register any change looking to available markets data.

Figure 21 | Government Securities Yield Curves (January 2020-December 2020)



Source: Author's Calculations.

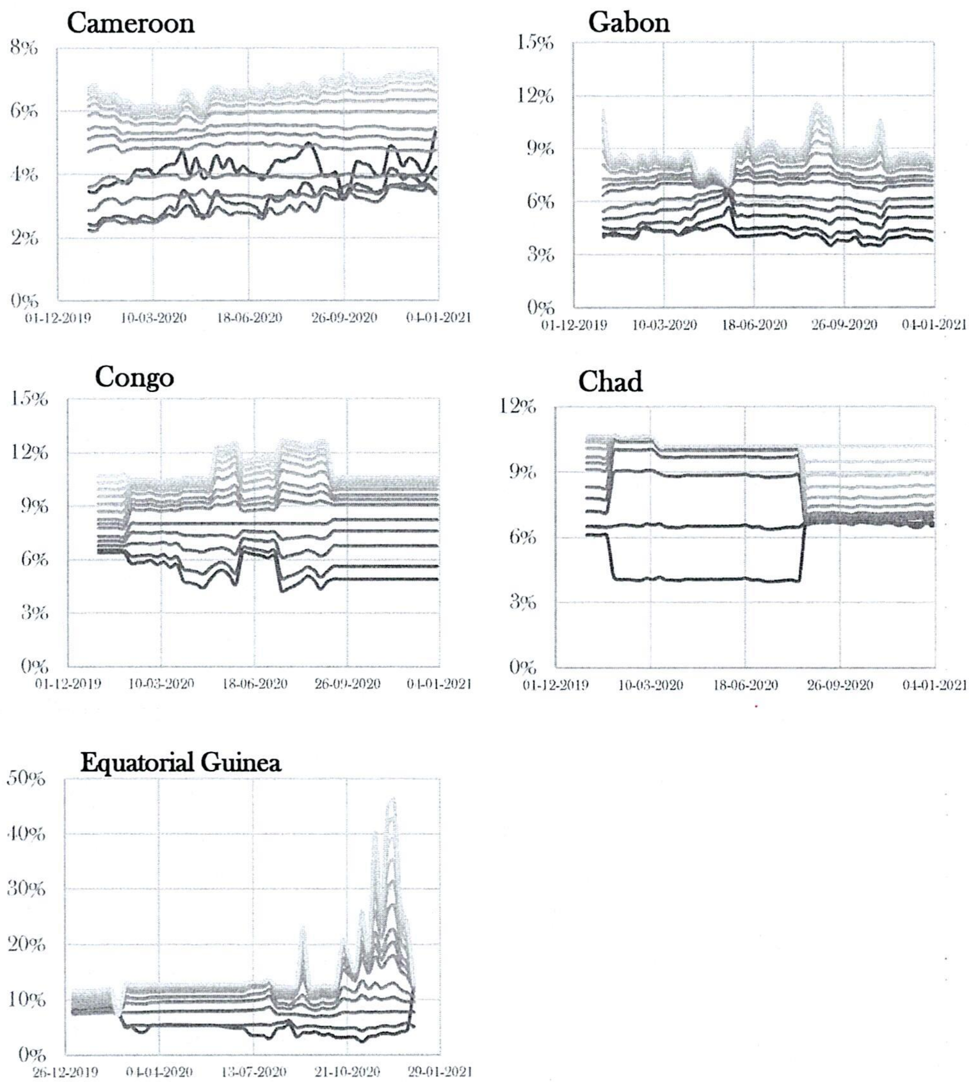
Figure 22 shows the dynamic evolution of yields for reference maturities including 3 months, 6 months, 1 year, 1.5 year, 2 years, 3 years, 3.5 years, 4 years, 5 years, 6 years, 7 years, 8 years, 9 years, and 10 years.

Even if the shape of the yield curve remains stable across time, we can see that short-term securities are highly volatile in Cameroon. We can also notice that there is no hierarchy in the short area. Moreover, it appears that long and short rates are weekly correlated.

By contrast, it appears that short, medium and long dated securities behave in the same way in Equatorial Guinea. Yields tend to explode at the last quarter of 2020.

The movements of yields appear to be strongly positively correlated in Chad. This can be explained by the fact that the country do not use to resort to the government securities market via subscription. Therefore, the yield curve tends to reproduce its trajectory across time. The correlation is also strong in Gabon. However, it is negative between short and long maturities, positive among short maturities and positive among long maturities.

Figure 22 | Evolution of Extracted Weekly Spot Rates for Fixed Maturities.



Source: Author's Calculations
Note: The lines have unique colors from blue shades for the shortest maturities to white for the longest maturities.

This chapter aims at constructing weekly yield curves of CEMAC countries. For that purpose, we employ market data that consist on issues and transactions of bond auctions, T-Bills, repo and repo reverse. The data covers the period August 01, 2019 to December 31, 2020. The methodological framework relies on parsimonious models including the canonical Nelson and Siegel model and the extended version of Svensson.

The empirical result shows that risk-free rate curves obtained are increasing, with positive slopes and the forward curve is above the spot curve. The fitting yield curves present significant price errors for short dated maturities in Cameroon, they are pronounced for medium maturities in Congo and significant for long-term maturities in Gabon. In a dynamic approach, the shape of the Cameroon's yield curve does not significantly change across time suggesting that the price risk is relatively low for these securities. By contrast, the shape of the term structure in Gabon and Equatorial Guinea significantly change across weeks. This erratic movement suggests that the interest rate risk is very pronounced for these securities.

In Cameroon, short-term securities are highly volatile and there is no hierarchy among yield in the short area of the curve. However, it appears long and short rates are weekly correlated. By contrast, it appears that short, medium and long dated securities behave in the same way in Equatorial Guinea and the yields explode at the last quarter in 200. Yields are significantly positively correlated in Chad. The correlation is also strong in Gabon. However, it is negative between short and long maturities, positive among short maturities and positive among long maturities.

As implications, mutual funds must limit their exposure to Gabon, Congo and Equatorial Guinea when they anticipate an increase of interest rates. In contrast, fixed income portfolio managers in the CEMAC zone may increase they exposure to these securities in a context of interest rate decrease. For risk averse markets participants, it is suitable to invest on Cameroon government securities which presents the lowest level of interest rate risks. They also provide the lowest yields to investors. As a result, they may invest on long dated maturities to expect high returns for their fixed income portfolio. Another suitable strategy for life insurance companies can consist on a combination of long-term securities in Cameroon and short dated maturities in Congo, Gabon and Equatorial Guinea in order to build a duration hedged portfolio. Government securities in Chad and Central African Republic appear to be a source of diversification since they do not have a specific behavior. Additionally, these countries do not use to resort to the regional public securities market.

Chapter IV| Bond Mutual Fund Valuation with Yield Curves

Much of the analysis and pricing activity in fixed income markets revolves around the yield curve. More specifically, the term structure indicates bonds that are cheap or dear to the curve. Placing bonds relative to the zero-coupon yield curve helps to highlight which bonds should be bought or sold either outright or as part of a bond spread trade.

The main goal of this chapter is to make a valuation of a bond mutual fund using extracted yield curves in the chapter III. We also make a valuation of this bond mutual fund using regulatory yield curves published by the central bank of central African states. For that purpose, we employ data from CRBC-PROSPERITE which is bond mutual fund. Data are extracted from MANAR Market which is the enterprise resource planning employed by ASCA Asset Management. This chapter examines the sensitivity of the bond portfolio to valuation methods based on yield curves: regulatory yield curves and constructed yield curves in the previous chapter. This allows us to make a comparative analysis between the valuation obtained using weekly yield curves and the valuation using BEAC yield curves.

The rest of the chapter is organized as follows. The next section presents the regulatory and the financial framework of mutual funds in the CEMAC zone. Section 2 presents some summary statistics and provides preliminary analyses of the bond mutual fund. More specifically, we depict its financial statements and its sensitivity to an interest rate shock. The third section of this chapter makes a sensitivity analysis by examining the valuation differences. The last section conducts a quantitative risk analysis of valuation differences.

1. Regulatory and Financial Framework of Mutual Funds

Mutual funds in the CEMAC zone are set by regulations that fixed applicable obligations and rules. In contrast, the financial framework is not well defined but each asset management company can determine accounting rules and methods as well as valuation methods.

1.1. Regulatory Framework of Mutual Funds

The regulatory framework of mutual funds is mainly set by the COSUMAF general regulations enacted in 2008 which is the primary applicable act. This regulatory act indicates that the registration of mutual funds is compulsory. The structure and creation of mutual funds, appointment of managers and investors, investment restrictions, compliance and penalties are all defined under COSUMAF Regulations. The regulatory framework of mutual funds is also guided by a battery of regulatory acts that states their classification and pricing associated to their management and interests conventions.

Instruction n° 03-11 enacted in May 16, 2011 makes a classification based on asset classes of undertakings for collective investment in transferable securities. The regulation allows us to distinguish: equity funds, debt funds, money funds, and hybrid funds. Equity funds are primarily invested in stocks, which represents more than 60 percent of the mutual fund. These funds invest the money collected from different investors into shares of different firms or other mutual funds. The performance of these shares determines the returns or losses in the market. These funds come with quick growth. So, the risk is comparatively higher in investing money in these funds. Debt funds mainly invest (more than 60 percent in fixed-income securities which a tenor greater 1 year. Money market funds are destined to investors who operate money on the cash market. Usually, they are run by the government, corporations or banks by issuing securities like T-bills. The fund manager invests the money and disburses regular dividends in return. They are good for short-term plans and are relatively less risky. Hybrid Funds are also known by the name Balanced Funds. They are a mix of bonds and stocks, thereby, bridging the gap between debt funds and Equity Funds. There is no fixed ratio. It may be variable or fixed. They are appropriate for investors looking for high yields and risks rather than holding a lower but regular income.

Resolution n° 11 enacted the Ministerial Committee of the monetary union in April 30, 2020 states the pricing of mutual funds' management. This regulatory act set the maximum prices applicable to management fees, placement fees of foreign undertakings for collective investment in transferable securities, subscriptions and redemptions fees, outperformance payments, agreement fees and other operational fees.

Interest's conventions are set by the methodological note on the building of government securities yield curve in the CEMAC zone. The note determines the characteristics of public fixed income instruments traded on the CEMAC zone (bond syndication, bond adjudication, and T-Bills). On the whole, we can retain that interests are prepaid for T-Bills with an Exact/360 base. By contrast, interest are postpaid and computed with an Exact/365 base.

The regulatory bodies are formed by the Financial Market Supervisory Commission of Central Africa (COSUMAF hereafter), the central bank of central African states (BEAC), Asset Management Companies (AMCs), and Custodians. COSUMAF is the apex regulator of capital markets in the monetary union of central Africa. This market authority is also authorized to periodically inspect mutual fund organizations to ensure compliance with regulations. COSUMAF also regulates other fund constituents such as asset manager companies (AMCs), brokers, and custodians. BEAC is the monetary authority of the CEMAC zone and is also the regulator of the banking system. BEAC is the regulator of the regional government securities market and also the regulator of money market. Mutual funds that invest in these securities are then affected by the BEAC regulations on the structure pricing and trading of these instruments.

1.2. Financial Framework of Mutual Funds

The market authority has not yet set the financial framework of mutual funds. Each AMC must define an accounting plan. However, there are common rules for valuing mutual funds among practitioners.

Accounting rules and methods must respect Generally Accepted Accounting Principles (GAAP), including accrual principle, conservatism principle, consistency principle, cost principle, full disclosure principle, matching principle, materiality principle, monetary unit principle, reliability principle, revenue recognition principle, and time period principle.

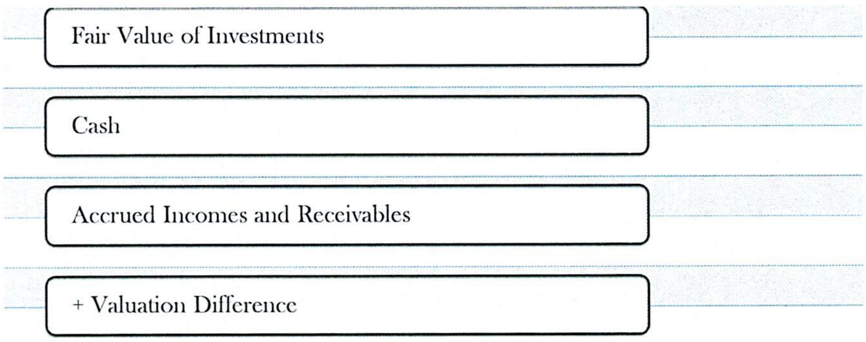
Accounting record of investment income can be achieved using two methods accrued coupon or paid coupon method. The paid coupon method suggests to attach interests to investment income when coupon payments take place. Accrued coupons are considered as valuation difference are not separately recorded. Accrued coupon method suggests to record earned interests and principal.

The value of a fund is determined by its net asset value (NAV), which is computed by the difference between the total value of assets and the total value of liabilities. The fund under consideration is a bond mutual fund in accordance with the act n°03-11 relating to the classification of undertakings for collective investment in transferable securities.

The assets of a mutual fund include the total market value of the fund's investments, cash and cash equivalents, receivables and accrued income. The market value of the fund is computed once per week according a valuation methodology. Bonds and similar debt securities are often valuing using three approaches: market value, historical price and present value. Many practitioners employ closing prices or average of the three last closing prices when securities are quoted and traded. Historical price is largely employed in the CEMAC zone. This approach suggests that the value of a debt instrument is equal to the nominal value plus accrued interests. The present value is not yet expanded among CEMAC countries. This methodology is used when the security has not recorded any transactions and the historical price does not correspond to markets and economic outlook. The valuation then relies on an actuarial method based on the yield curve.

Since a fund may have a certain amount of capital in the form of cash and liquid assets, that fraction is accounted for under the cash and cash equivalents heading. Receivables include items such as dividends or interest payments applicable on that day, while accrued income refers to money that is earned by a fund but yet to be received. The sum of all these items constitute the fund's assets (see figure 23).

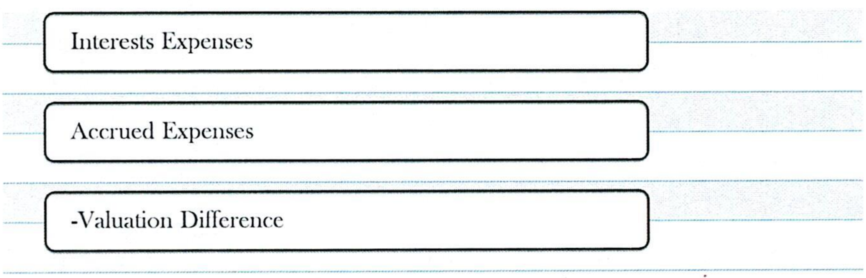
Figure 23 | Asset Components



Source: Author.

The liabilities of a mutual fund typically include money owed to the lending banks, pending payments and a variety of charges and fees owed to various associated entities (see figure 24). Additionally, a fund may have foreign liabilities that may be the shares issued to non-residents, income or dividend for which payments are pending to non-residents, and sale proceeds pending repatriation. All such outflows may be classified as long-term and short-term liabilities, depending upon the payment horizon. The liabilities of a fund also include accrued expenses, like staff salaries, utilities, operating expenses, management expenses, distribution and marketing expenses, transfer agent fees, custodian and audit fees, and other operational expenses.

Figure 24 | Liabilities Components



Source: Author.

For the period under consideration, the balance sheet of the mutual fund was mainly designed as follows:

Table 6 | **Balance Sheet of the Fund**

Assets	Liabilities
Investments	**Interests**
T-Bonds	**Accrued Expenses**
T-Bills	
Cash	

Source: Mutual Fund Financial Statements.

The Net assets of a scheme will go up if the amount of assets increase. More specifically, an increase is recorded whenever investors buy additional units in the scheme and bring in funds, or when the value of the investments held in the portfolio goes up, or when the securities held in the portfolio earns income such as dividends from shares or interest on bonds held. By contrast, the net assets of the scheme will go down when liabilities increase. if investors take out their investments from the scheme by redeeming their units or if the securities held in the portfolio fall in value or when expenses related to the scheme are accounted for. The net assets of the scheme are therefore not a fixed value but keep changing with a change in any of the above factors.

A mutual fund works by collecting money from a large number of investors. It then uses the collected capital to invest in a variety of stocks and other financial securities that fit the investment objective of the fund. Each investor gets a specified number of shares in proportion to their invested amount, and they are free to sell their fund shares at a later date and pocket the profit/loss. Since regular buying and selling of fund shares start after the launch of the fund, a mechanism is required to price the shares of the fund. This pricing mechanism is based on net asset value per unit.

NAV of a share is the market value of the securities held by the scheme. In the CEMAC zone, NAV of a share is weekly computed to determine the value of a mutual fund. The formula for a mutual fund's NAV calculation is straightforward:

$$\text{NAV of a share} = (\text{Assets} - \text{Liabilities}) \,/\, \text{Total number of outstanding shares} \qquad (45).$$

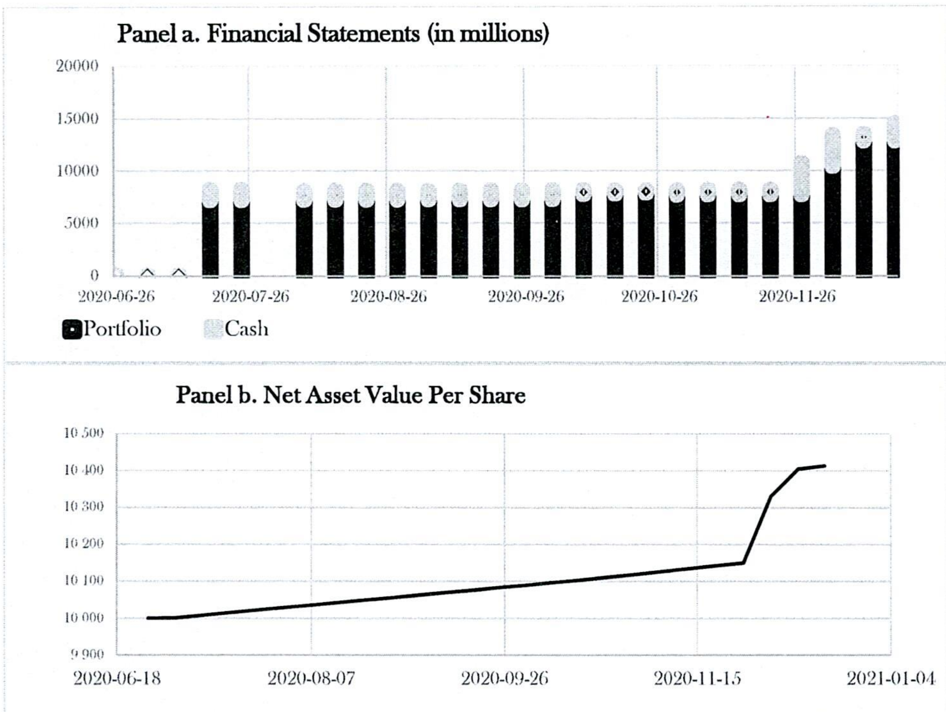
2. Descriptive Analyses and Preliminary Analyses

This section aims to make descriptive analyses of financial statements of the Fund. We examine the evolution of the net asset value and its components. We also make an overview of the main investments. Finally, we conduct a risk analysis of the Fund. For that purpose, we use weekly data extracted from MANAR which is a specialized software dedicated to undertakings for collective investments in transferable instruments (UCITS). Data range from July 03 to December 18, 2020. Variables include: the International Securities Identification Number (ISIN); the facial rate; the name of Instrument, quantity of securities, net asset value, number of shares, subscription or redemption, and available cash.

2.1. Net Asset and Main Investments

The net asset value and performance evolve according to subscriptions, accrued interest and investment decisions. Three main subscriptions have been registered in 2020: July 17, November 27, and December 04; leading to a structural break of the net asset (see Figure 25, Panel a). We can also see in Figure 25, Panel b that the asset value per share has recorded an impressive upward and the fund has observed a peak of performance in November 27, 2020. Indeed, the Fund has generated important accrued interests and the asset value of the fund has significantly raised after that XAF 2 696 000 697 were invested on the regional government securities market.

Figure 25 | Evolution of Net Asset



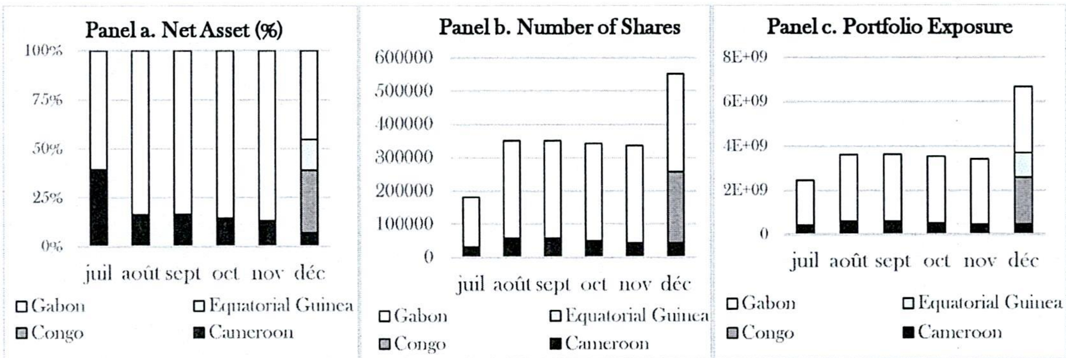
Source: Author's Calculations

In accordance with the management mandate, the active position to cash is often under 20 percent of asset, except in November 27, 2020 and December 04, 2020.

For the period under consideration, the Fund has only invested in sovereign debt securities. The geographic composition in figure 26 lists the countries where the Fund invests. Figure 26, Panel a. suggests that the market value of the Fund is mainly driven by investments in Gabon. The net asset also depends to investments in Cameroon even though its contribution has decreased across time.

The portfolio is mainly formed by Gabon securities which represents on average more than 60 percent of the Fund (see Figure 26, Panel b). The exposure is also highest to Gabon government securities although Figure 26, Panel c. shows that the Fund has invested an important amount in Congo and Equatorial Guinea in December 2020.

Figure 26 | Geographic Composition of the Fund



Source : Author's Calculations.

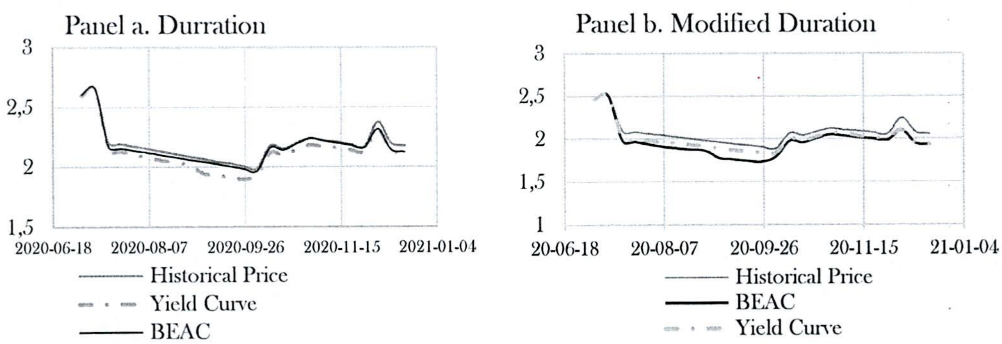
However, the fund composition was in adequacy with geographic requirements admitted by the mandate management which imposed that 50 percent was invested in Cameroon and 50 percent in Gabon.

2.2. Fund Sensitivity

In this subsection, we examine the interest rate sensitivity of the mutual fund using the Macauley and the modified duration.

Figure 27 outlines the evolution of the duration of the Fund between July and December 2020. The portfolio strategy relies on a relative constant effective average maturity, ranging from 1.99 to 2.66 years (Figure 27, Panel a).

Figure 27 | Mutual Fund Duration (July 03 - December 04, 2020)



Source: Author's Calculations.

Summary statistics reported in Table 7 shows that the interest rate sensitivity of the Fund is most pronounced on average using the historical price which assume that the term structure is flat. This is not surprising since yields equal to coupon rates and lower than yields extracted from regulatory and weekly curves. The duration volatility is highest for extracted weekly yield curves.

Table 7 | Summary Statistics for Duration

	Historical Price	BEAC	Extracted Yield Curves
Panel a. Duration			
Mean	2.19	2.17	2.13
Median	2.18	2.15	2.13
Max	2.66	2.66	2.65
Min	1.99	1.97	1.90
Std. Dev.	0.16	0.17	0.18
Skewness	1.66	1.75	1.52
Kurtosis	5.69	6.00	5.46
Panel b. Modified Duration			
Mean	2.08	1.97	2.01
Median	2.06	1.96	1.99
Max	2.51	2.51	2.51
Min	1.89	1.73	1.84
Std. Dev.	0.15	0.19	0.16
Skewness	1.67	1.52	2.00
Kurtosis	5.74	5.54	6.76

Source: Author's Calculations.

Considering non-flat curves, we can notice that if interest rates rise instantaneously by 1 percent, the bond fund is thus expected to lose 1.97 or 2.01 percent of its value on average based on its modified duration (see Panel b, Table 7), using BEAC and computed yield curves respectively.

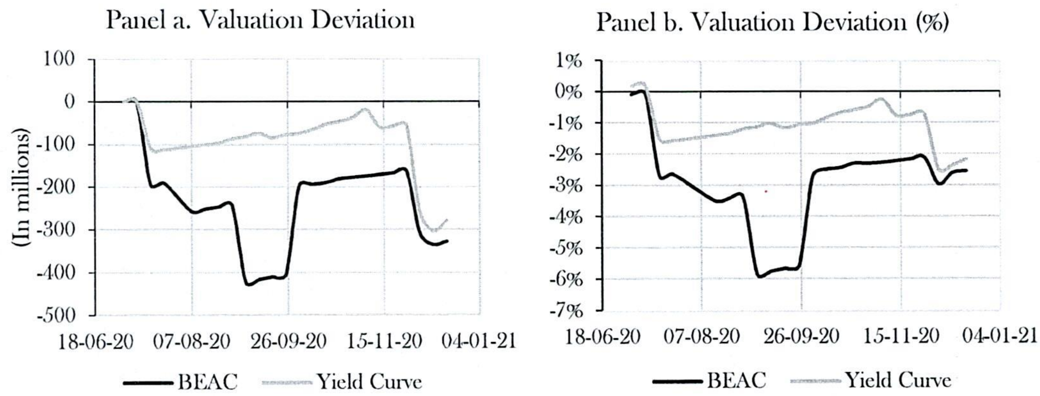
3. Valuation Difference Analysis

This section aims at valuing the mutual fund using BEAC yield curves as specified in the management mandate. We also make a valuation based on weekly yield curves that we have extracted in the previous chapter. Analyses focus on the impact on the net asset value and the performance of the Fund.

3.1. Impact of Valuation Methodologies on the Net Asset Value

Figure 28 below shows the valuation difference which appears when we make a valuation of the Fund using regulatory yield curves and weekly curves extracted in the previous chapter. We can see that the valuation at historical cost overvalues the portfolio compared to a valuation based on the actuarial method with monthly yield curves published by the BEAC or weekly extracted yield curves in the previous chapter (see Figure 28, Panel a). The persistent negative difference stems from the Fund's high exposure to Gabonese government securities for which investors often require high yields when issuing or trading on the secondary market. More specifically, the Gabon Treasury usually offers high coupon rates and discounts recorded on the primary and secondary market are significant. The gap has worsened from November 2020 with the fund's overexposure to Gabon and Congo's government public securities. Figure 28, Panel b shows that the deviation exeeds 2 percent.

Figure 28 | Valuation Differences



Source: Author's calculations.

The valuation deviation is always more pronounced when we employ BEAC yield curves than when we use our weekly yield curves. This is not surprising insofar as monthly regulatory yield curves are not suitable with the weekly calculation of the net asset per share. Moreover, a weekly valuation for a given month is achieved by referring to the yield curve of the previous month. The gap is highly pronounced in September because the valuation based on the BEAC is not suitable with the 10 year T-Bonds held by the Fund. Indeed, this security offer a coupon rate equals to 7 percent while the regulatory yield curve is under 6.6 percent. This automatically leads to a significant methodology deviation. Therefore, this valuation leads to significant differences which can overpass XAF 400 million.

By contrast, the valuation through extracted yield curves initially leads to a positive difference in July 3 and 10, 2020. Although the difference also remains negative for the rest of the sample, the methodology deviation does not exceed 1 percent, except for the period ranging from November to December 2020.

The statistics associated to the methodology deviation are summarized in table 8 below. On average, valuation based on weekly yield curves provide the lower difference (-1.03 percent) with the historical price approach. This methodology also leads to positive difference (0.18 percent) while maximum for the BEAC yield curves is null. The results also confirm that the methodology deviation is less volatile using extracted yield curves (0.70 percent).

Table 8 | Summary Statistics of Portfolio Valuation Differences

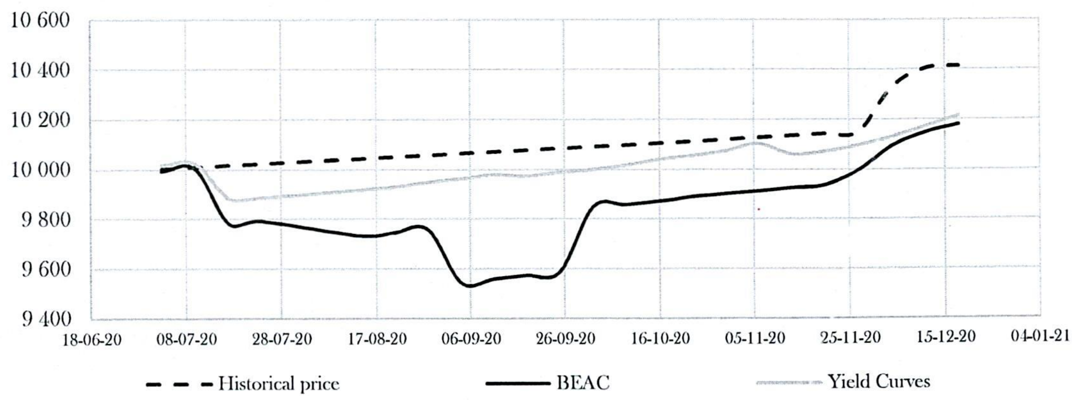
	BEAC Curves	Extracted Yield Curves
Mean	-2.86%	-1.03%
Median	-2.61%	-1.03%
Maximum	0.00%	0.18%
Minimum	-5.85%	-2.50%
Std. Dev.	1.59%	0.70%
Skewness	-0.28	-0.25
Kurtosis	3.10	2.85
Jarque-Bera	0.34	0.29
Probability	0.84	0.87

Source: Author's Calculations.

3.2. Impact of Valuation Methodologies on the Performance

Figure 29 shows that valuing the bond mutual fund using yield curves reduces the net asset per share. However, we can notice that there is always an upward dynamics especially when using weekly yield curves. The subscription recorded in November 27, 2020 does not have an impact on the net asset per share valuation using yield curves.

Figure 29 | Evolution of the Net Asset per Share

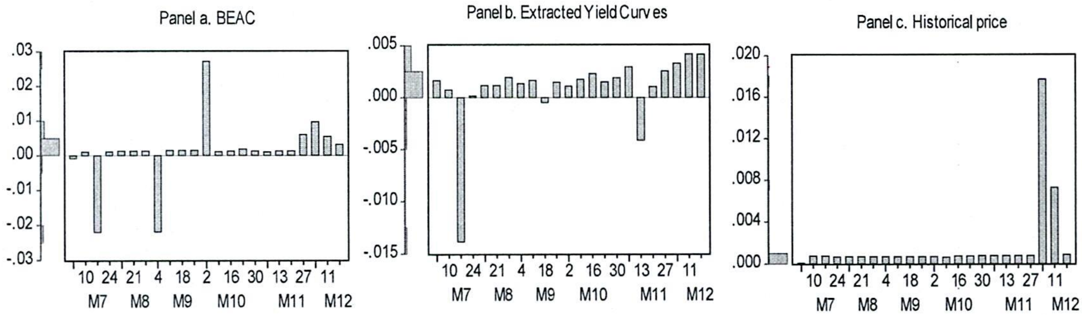


Source: Author's Calculations.

The valuation using yield curves allows us to examine the performance of the Fund when valuing with yield curves. Using an historical price valuation, Figure 31, Panel a shows that the Fund only records a positive performance drawn by accrued interests associated to amounts invested since the company has adopted a hold and buy strategy.

Following the BEAC valuation methodology, we can notice that the performance has collapsed by 2.2 percent in July 17 and September 04, 2020. By contrast, the Fund has recorded a peak of performance in October 02, 2020, with the net asset per share increasing by 2.7 percent. The performance also collapses in July 17, 2020 when we employ our extracted yield curves for valuing the Fund.

Figure 30 | Impact of Valuation Methodologies on the Performance



Source: Author's Calculations.

Table 11 displays some summary statistics of the Fund's performance. Valuation from our extracted weekly yield curves displays the lowest performance and presents the lowest volatility. The maximum performance recorded within this valuation is 0.42 percent while the valuation based on the BEAC yield curves records a peak of performance equals to 2.72 percent.

Table 9 | Summary Statistics on the Fund Performance

	BEAC Curves	Extracted Yield Curves	Historical Price
Mean	0.11%	0.08%	0.17%
Median	0.13%	0.15%	0.07%
Maximum	2.72%	0.42%	1.77%
Minimum	-2.21%	-1.39%	0.01%
Std. Dev.	0.92%	0.36%	0.38%
Skewness	-0.37	-3.16	3.74
Kurtosis	6.81	13.37	15.97
Jarque-Bera	14.47	141.43	214.71
Probability	0.00	0.00	0.00

Source: Author's Calculations.

4. Empirical Characterization of Valuation Difference Risk

After some preliminary analyses and the assessment of valuation differences, this section conduct a quantitative risk assessment. More specifically, we measure the valuation difference risk of the Fund. The section is organized in two paragraphs. First, we present quantitative tools that we employ in our methodological framework. Second we present and discuss the main empirical results.

4.1. Methodological Framework

Our methodological framework is structured into two stairs. First, we employ data generators for profits and loss instead of economic scenario generators (ESG). Second, we use two risk measures including the value at risk (VaR) and the expected shortfall (ES).

4.1.1. Data Generator for profit and losses (P&L)

In this study, two tools are employed to generate P&L data. On the one hand, we use the use normal distribution to generate P&L. On the other hand we employ an autoregressive process to generate data.

If a random variable, x, follows a normal distribution the form of its probability density function is given by:

$$f(x) = \frac{1}{2\sigma\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{46}$$

where μ is the mean or expectation of the distribution while the parameter σ is the standard deviation.

In a different range, a random variable, X_t, which follows an $\text{ARIMA}(p,d,q)$ process can be written as:

$$\left(1-\sum_{i=1}^{p} a_i L^i\right)(1-L)^d X_t = \left(1+\sum_{i=1}^{p} b_i L^i\right)\varepsilon_t \tag{47}$$

with a_i and b_i being parameters and $\varepsilon_t \sim \aleph(0,1)$. L is a lag operator.

4.1.2. Risk Measures

The present study only explores the VaR and the ES. For a tolerance level $\alpha \in [0,1]$, the value at risk is the quantile:

$$\text{VaR}_\alpha(X) = \inf\{x \in \mathbb{R} : F(x) \geq \alpha\}, \tag{48}$$

which means find the greatest lower bound of loss x such that the cumulative probability of x is greater than or equal to α. For example, if α equals to 95 percent. This would mean that a decision maker would not tolerate loss in more than 5% of all risk scenarios.

There are three VaR Approaches: parametric VaR, historical VaR, and Monte Carlo (MC) VaR. In this research, we only perform parametric VaR using an ARIMA(1,1,0) and a Monte Carlo VaR through the normal distribution or mixture normal distributions.

Using the VaR_α definition we can also define ES_α as:

$$\mathrm{ES}_\alpha = \mathrm{E}\left[X \middle| X \geq \mathrm{VaR}_\alpha\right] \tag{49}$$

The expected shortfall is the average of all losses greater than the loss at a VaR_α associated to probability α and $\mathrm{ES} \geq \mathrm{VaR}$.

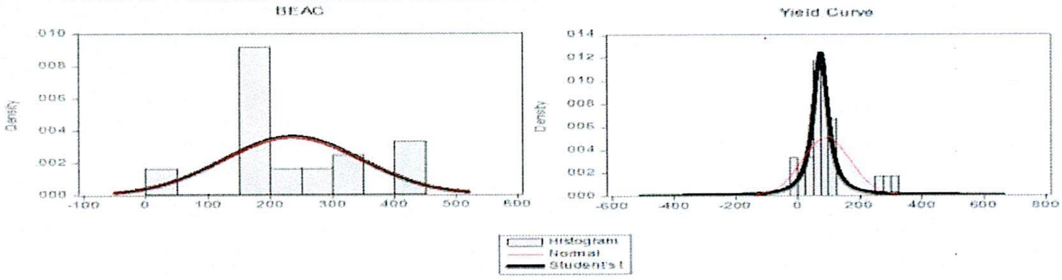
4.2. Empirical Results

After presenting the impact of methodology deviation on the net asset value and the performance of the Fund, we can now quantify the risk of a valuation difference. More specifically, we examine the uncertainty associated with potential changes in the value of the mutual fund caused by changes in interest rate levels.

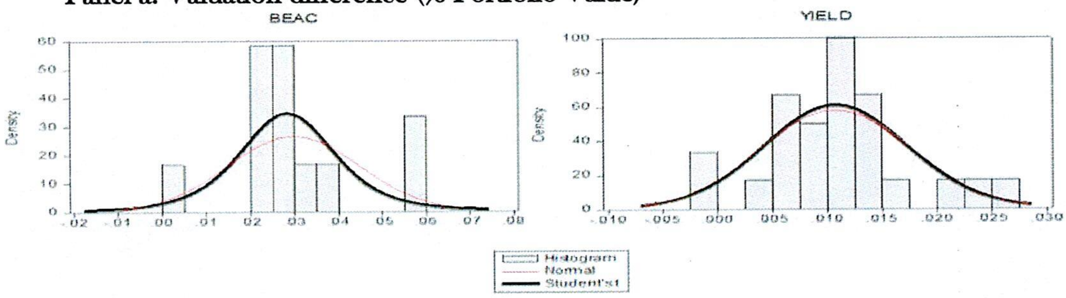
Figure 31 below depicts the distribution of valuation difference and the theoretical normal and student distributions.

Figure 31 | Valuation Difference Density

Panel a. Valuation difference in millions



Panel a. Valuation difference (% Portfolio Value)



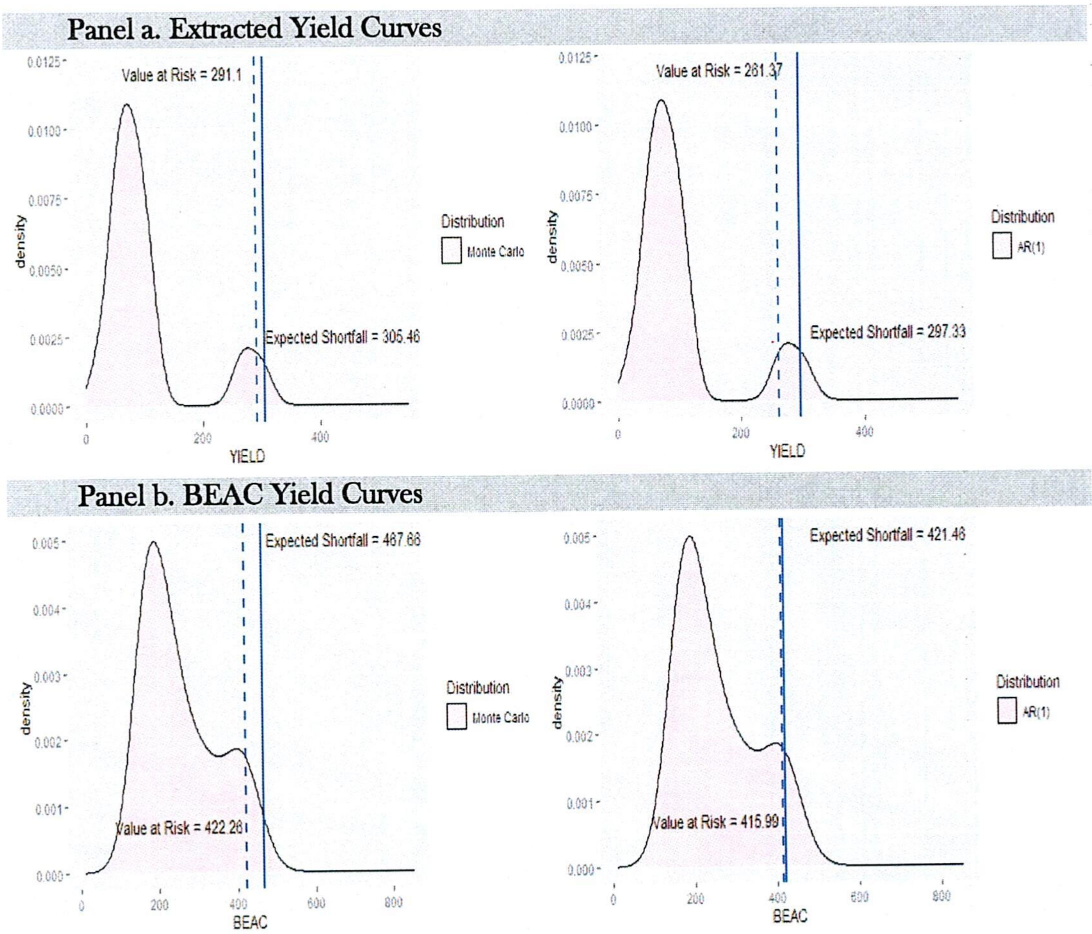
Source: Author's Calculations.

We can see that the normal and student distributions have a good fitting of the valuation difference obtained with BEAC yield curves. By contrast, it appears that the distribution of valuation difference obtained using extracted yield curves present fat tails which are well fitted by the student distribution.

Looking to the distribution valuation difference in percentage of portfolio value, it appears that there are fat tails in our distributions. Therefore, the student distribution is more appropriate to fit our series. Instead of a student distribution, we employ a mixture of normal distributions to take tails into consideration

Results in figure 32 shows the XAF VaR results. We can notice that VaR does not significantly vary across methodologies. Using weekly yield curves, the Monte Carlo value at risk accounts for XAF 291.1 million while predictions with an autoregressive process lead to a VaR equals to XAF 261.3 million.

Figure 32 | XAF VaR of Valuation Difference (in Millions)



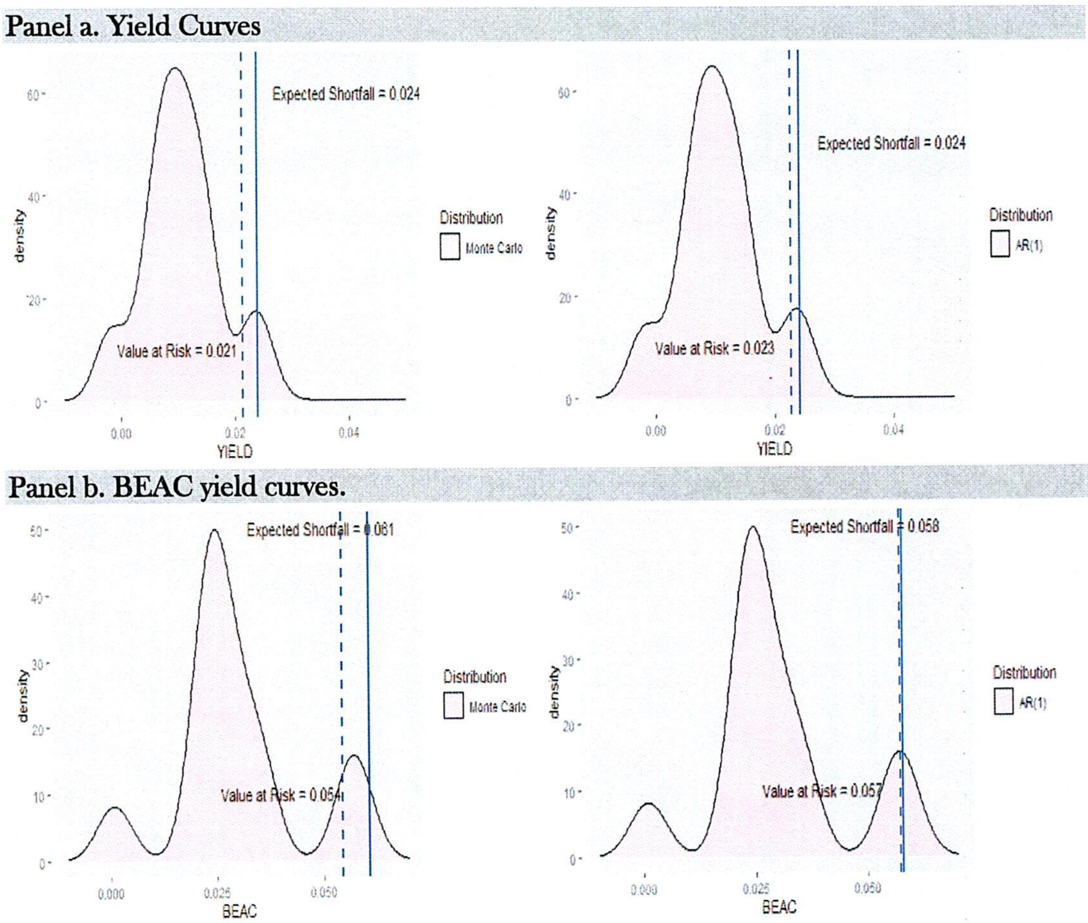
Author's Calculations.

The mean of valuation difference is XAF 93.79 and 234.78 million for weekly yield curves and BEAC yield curves respectively. Therefore, basing on weekly yield curves, the Fund could keep around XAF 200 million to face erratic changes of interest rates that lead to a significant negative valuation difference. In the same vein, the economic capital must be equal to around XAF 190 million considering BEAC curves.

The value at risk of valuation difference decreases when we employ make a parametric computation to model valuation difference. It decreases by around XAF 40 million (from 467 to 421 million) according to BEAC yield curves while the decrease is lesser, about 30 millions, when we employ our yield curves (from 291 to 261 millions).

Figure 33 depicts the VaR of valuation differences as percentage of the portfolio value.

Figure 33 | VaR of Valuation Difference (% Portfolio Value)



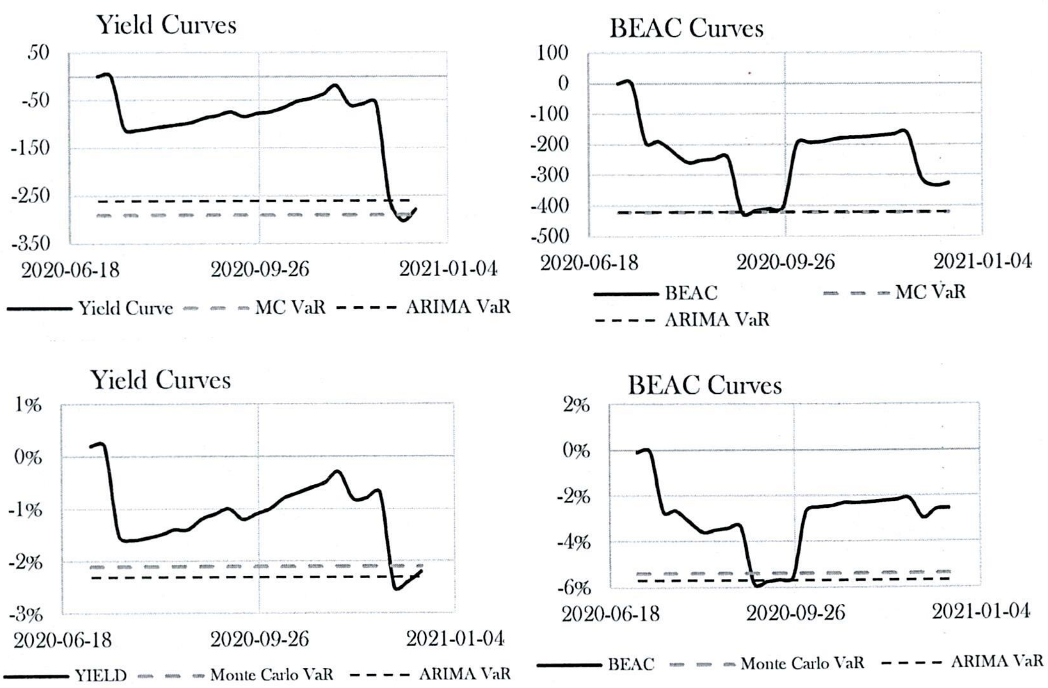
Source: Author's Calculations

Looking to the MC VaR of valuation difference as a percentage of portfolio value, we can see that the fund may register a valuation difference equals to 2.1 percent of its market value at a confidence level of 5 percent using our yield curves. However, pricing with BEAC curves suggests that the Fund may register a loss that represents 5.04 percent of the portfolio market value at 5 percent confidence level. Results relatively stable even when we conduct a parametric VaR computation with an autoregressive process.

The gap between mean and VaR represents 1.3 and 4.3 percent, respectively for our curves and BEAC curves. Therefore, the economic capital to face an excessive valuation difference is around 1.3 percent of investments if we employ weekly curves to quote bonds. However, economic capital must be greater if the valuation is achieved with regulatory yield curves.

After computing VaR, it is important to achieve backtesting which measures the accuracy of the value at risk calculations. Figure xxx depicts daily profits and losses (P&L's) against (negative) value-at-risk for an actual trading portfolio. It provides a simple graphical analysis of how well the Fund's value-at-risk measure performed. The backtesting shows that our VaR calculations are good. Breaches are lower than 0 for each VaR computation. In other word, less than 4 loss are greater than computed VaRs.

Figure 34| VaR Backtesting



Source: Author's Calculations.
Note: The lines running across the bottom of the chart indicates the portfolio's (negative) one-day 95% XAF and %value-at-risk.

This chapter aims to make a valuation of a bond portfolio using yield curves. For that purpose we exploit financial statements of a bond mutual fund ranging from July 03 to December 18, 2020. We make a valuation of bond using the historical price methodology, regulatory yield curves constructed by the central bank of central states and weekly yield curves constructed in the previous chapter. Finally, we conduct an analysis of valuation difference. More specifically, we examine the impact on the net asset value and the performance of the Fund. The main results suggest that the valuation of debt securities at historical cost overvalues the portfolio compared to a valuation based on the actuarial method with monthly yield curves published by the BEAC or weekly yield curves constructed in the previous chapter. However, the valuation difference is more pronounced for valuation based on BEAC yield curves. This is explained by the fact that regulatory yield curves are monthly available while the valuation of mutual funds is achieved weekly. Therefore, valuing with BEAC curves suggests that one refers to the yield curves of the past month and generates significant valuation differences.

The empirical results have interesting implications. BEAC yield curves must be available weekly or the Fund can employ an internal model to determine the present value of bonds. Moreover, Nelson and Siegel parameters must be communicated to AMUs to make a precise extraction of yields for intermediary tenors. As implications for portfolio management, the Fund should limit its exposure to Gabonese and Congo government securities which registers an important interest rate risk. Another alternative strategy can consist to limit duration risk.

Concluding Remarks and Implications

There has been an increased use of local bond markets in African countries, mainly after the 2008 global financial turmoil which has led to the development of asset management activity. In the CEMAC zone existing mutual funds are mainly invested in fixed income securities markets. Their valuation is not yet guided by a regulation of the. Without a regulatory framework, practitioners consider that the value of a fixed income asset is the sum of the nominal value and the accrued coupon not yet due.

The main goal of this dissertation is to make a sensitivity analysis of bond mutual fund to the yield curve. More exactly, this study aims at investigate the impact of the term structure on the valuation of a bond mutual fund within the context of CEMAC countries.

For that purpose, we extract weekly yield curves of CEMAC countries using market data of the primary and secondary market of the regional government securities market. We also employ data on the interbank repurchase agreements to fit the short area of the yield curve. We also make a comparative analysis of valuation differences which appear using the BEAC yield curves and those which appear using our extracted yield curves. Finally, we examine the impact of these valuation differences on the net asset value and the performance of the fund.

Fitting our yield curves for the period covering August 01, 2019 to December 2020, we find that risk-free rate curves of CEMAC countries display a positive sloping across maturities. As a result, forward rates are often higher than spot rates. The level factor is greater than 3.5 percent for all CEMAC countries. This result is consistent with the TIAO offered by the central bank during the period under consideration. However, some significant heterogeneities appear among CEMAC countries. The fitting yield curves present significant price errors for short dated maturities in Cameroon, they are pronounced for medium maturities in Congo and significant for long-term maturities in Gabon. Results also show that the evolution of the yield curve is relatively stable is Cameroon while they explode in Gabon, Equatorial Guinea and Congo. This is suitable with illiquidity and the last liquid point observed in these markets which does not exceed 8 years. The dynamics of yield curves across time also reveals that interest rates are not well correlated.

Empirical results show that valuing bond mutual fun at historical cost overvalues the portfolio compared to a valuation based on the actuarial method with monthly yield curves published by the BEAC or weekly exctracted yield curves. The negative difference stems from the Fund's high exposure to Gabon and Congo's government securities for which investors often require high yields when issuing or trading on the secondary market. The valuation deviation is always more pronounced when we employ BEAC yield curves than when we use our weekly yield curves. This is not surprising insofar as monthly regulatory yield curves are not suitable with the weekly calculation of the net asset per share. Valuing the bond mutual fund using yield curves also reduces the net asset per share of the Fund.

Using VaR and expected shortfall, we have found that valuing bond mutual with BEAC yield curves have more risks than valuing with weekly extracted yield curves. There are some heterogeneities using a parametric or a Monte Carlo VaR. The risk is more important with a MC VaR than a parametric VaR.

The empirical findings of this dissertation lead to significant implications for fixed income analysts.

First, bond mutual fund must limit their exposure to Gabon, Congo and Equatorial Guinea when they anticipate an increase of interest rates. In contrast, fixed income portfolio managers in the CEMAC zone may increase they exposure to these securities in a context of interest rate decrease.

Second, it is suitable for risk averse markets participants to invest on Cameroon government securities which presents the lowest level of interest rate risks. They also provide the lowest yields to investors. As a result, they may invest on long dated maturities to expect high returns for their fixed income portfolio. Another suitable strategy for life insurance companies can consist on a combination of long-term securities in Cameroon and short dated maturities in Congo, Gabon and Equatorial Guinea in order to build a duration hedged portfolio. Government securities in Chad and Central African Republic appear to be a source of diversification since they do not have a specific behavior. Additionally, these countries do not use to resort to the regional public securities market.

Third, BEAC yield curves must be available weekly or the Fund can employ an internal model to determine the present value of bonds. Moreover, Nelson and Siegel parameters must be communicated to AMUs to make a precise extraction of yields for intermediary tenors.

References

Anderson, N., Breedon, F., Deacon, M., Derry, A., and Murphy, G. (1997). *Estimating and Interpreting the Yield Curve*, Chichester: Wiley (originally published 1996).

Anderson, N. and Sleath, J. (2001). *New Estimates of the UK Real and Nominal Yield Curves*, Bank of England.

Aljinovic, Z., Poklepovic, T., Katalinic, K. (2012) Best fit model for yield curve estimation, *Croatian Operational Research Review* (CRORR), Vol. 3, 2012.

Bjork, T., and Christensen, B. J. (1999). Interest rate dynamics and consistent forward rate curves. *Mathematical Finance*, 9, 323–348.

Bliss, R. R. (1997) Testing term structure estimation methods, *Advances in Futures and Options Research*, 8, 197–231.

Bodie, Z., Kane, A., and Marcus, A. J. (1996). *Investments*, 3rd edition, Boston, Mass.: Irwin (originally published 1989).

Bolder, D. J. (2015). Fixed-Income Portfolio Analytics: A Practical Guide to Implementing, Monitoring and Understanding Fixed-Income Portfolios.

Bolder, D. and Stréliski, D. (1999) Yield Curve Modelling at the Bank of Canada, Bank of Canada, Technical Report no. 84, February.

Brousseau V. (2002). The functional form of yield curves, European Central Bank, Working Paper no 80.

Cairns, A. J. G. (1998). Descriptive bond-yield and forward-rate models for the British Government securities market, British Actuarial Journal, 4, 265–321.

Carriere, J. F. (1998) "Long-term yield rates for actuarial valuations," Actuarial File Library (online) http://nt80.syn.net/soa_lib/.

Chambers, D. R., Carleton, W. T., and Waldman, D. W. (1984) A new approach to estimation of the term structure of interest rates, *Journal of Financial and Quantitative Analysis,* 19, 3, 233–52

Choudhry, M. (2019). *Analysing and Interpreting the Yield Curve.* Chichester, UK: Wiley Finance Series.

Coleman, T. S., Fisher, L., and Ibbotson, R. G. (1992). Estimating the term structure of interest rates from data that include the prices of coupon bonds, *Journal of Fixed Income*, 2, 2, 85-116.

Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). *A Theory of the Term Structure of Interest Rates, Econometrica*, 53, 385–407.

Cox, J., Ingersoll, J. and Ross, S. (1981). A Re-examination of Traditional Hypotheses about the Term Structure of Interest Rates, Journal of Finance, 36, 769–799

Culbertson, J. (1957). The Term Structure of Interest Rates, Quarterly Journal of Economics, 71, 489–504.

Deacon, M. and Derry, A. (1994). Estimating the Term Structure of Interest Rates, *Bank of England Working Paper no. 24.*

Deacon, M. and Derry, A. (1994). Estimating the Term Structure of Interest Rates", *Bank of England Working Paper Series* No. 24, July 1994.

De Pooter, M. (2007). Examining the Nelson-Siegel class of term structure models: In-sample fit versus out-of-sample forecasting performance. Working Paper. Tinbergen Institute.
Diebold, F. X., Piazzesi, M., and Rudebusch, G. D. (2005). Modeling bond yields in finance and macroeconomics. *The American Economic Review*, 95, 415–420.
Diebold, F. X., and Li, C. (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics*, 130, 337–364.
Diebold, F. X., and Rudebusch, G. D. (2013). Yield curve modeling and forecasting: The dynamic NelsonSiegel approach. Princeton: Princeton University Press.
Duffie, D. (1992), *Dynamic Asset Pricing Theory*, Princeton University Press, 1992.
Dobbie, G. M. and Wilkie, A. D. (1979). The Financial Times–Actuaries Fixed Interest Indices, Transaction of the Faculty of Actuaries, 36, 203–13.
Douglas, L. G. (1988). Yield Curve Analysis: The Fundamentals of Risk and Return, New York: NYIF Corporation.
Fama, E.F., (1976). Forward rates as predictors of future spot interest rates, *Journal of Financial Economics,* 3, 4, 361–377.
Fama, E.F. and Bliss, R.R. (1987). The Information in Long-Maturity Forward Rates, American Economic Review, 77, 680-692.
Feldman, K. S., Bergman, B., Cairns, A. J. G., Chaplin, G. B., Gwilt, G. D., Lockyer, P. R., and Turley, F. B. (1998) Report of the Fixed-Interest Working Group (with discussion), *British Actuarial Journal*, 2, 4, 213–63 and 350–83.
Fisch, J. S. (1997) *Practical Introduction to Fixed Income Securities*, London: Euromoney
Fisher, I. (1896). *Appreciation and Interest*, Publications of the American Economic Association, 23–29, 88–92.
Gbongué, F. (2015). Un modèle de projection des taux sans risque dans la zone CIPRES. *Financial Afrik.*
Gbongué F., Planchet F. (2015). Analyse comparative des modèles de construction d'une courbe de taux sans risque dans la zone CIPRES, Bulletin Français d'Actuariat, 15, 30, p 129-168.
Gbongué, F., Planchet, F., Ahoussi, A. (2017). Proposition d'un modèle de projection des scénarios économiques pour le développement de la zone CIPRES, Assurances et gestion des risques, 84 (1-2).
Gbongué, F. (2019a). Amélioration de la méthodologie de construction de la courbe des taux sans risque dans la zone UEMOA. *Assurances et Gestion des Risques / Insurance and Risk Management*, 86(2-3), 127–164.
Gbongué F. (2019a) «Modélisation et prévision de la courbe des taux zéro-coupon pour le développement de la zone UEMOA», *Revue Économique et Monétaire*, No 25.
Gurkaynak, R. S., Sack, B., and Wright, J. H. (2007). The U.S. treasury yield curve: 1961 to the present. *Journal of Monetary Economics*, 54, 2291–2304
Hull J. and White A. (1990), Pricing interest-rate-derivative securities, *Review of Financial Studies,* 3, 573–592.
Kovachev Y., and Simeonov D., (2014). Yield Curve Fitting with Data from Sovereign Bonds, ISBN 978-954-8579-53-7, *Bulgarian National Bank.*
Lartey, V.C. and Li, Y. (2018). Zero-Coupon and Forward Yield Curves for Government of Ghana Bonds, *Sage Open,* July 2018.
Lartey VC, Li Y, Lartey HD, and Boadi, E.K (2019). Zero-Coupon, Forward, and Par Yield Curves for the Nigerian Bond Market. *Sage Open*, October 2019.

Litzenberger, R.H. and Rolfo, J. (1984). An International Study of Tax Effects on Government Bonds, *Journal of Finance*, 39, 1-22.

Lutz, F.A. (1940). The Structure of Interest Rates, *Quarterly Journal of Economics*, **55**, 36–63.

Manousopoulos, P., & Michalopoulos, M. (2009). Comparison of non-linear optimization algorithms for yield curve estimation. *European Journal of Operational Research*, 192, 594–602.

Martellini, L., Priaulet, P. and Priaulet, S. (2003). *Fixed-Income Securities: Valuation, Risk Management and Portfolio Strategies.* Wiley Finance.

McLeod, H. D. (1990). The development of a market yield curve: the South African solution, First AFIR International Colloquium, Paris, vol. 2, pp. 196–212.

Ronn, E.I. (1987). A New Linear Programming Approach to Bond Portfolio Management, *Journal of Financial and Quantitative Analysis*, 22, 439-466.

Rudebusch, G. D., and Wu, T. (2008). A macro-finance model of the term structure, monetary policy and the economy. *The Economic Journal*, 118, 906–926

McCulloch, J. H. (1971) Measuring the term structure of interest rates, *Journal of Business*, 44,1,19–31.

McCulloch, J. H. (1975). The tax-adjusted yield curve, *Journal of Finance*, 30, 3 811–30.

Mezui, A.M. Dynamics of Central Africa Regional Bond Market. *Africa economic brief.* 12, 7, 1-7.

Moungala W.P. (2013). Modélisation et gestion sur les marchés obligataires souverains. Thèse de Doctorat, Université de Rennes 1.

Nelson C.R. and Siegel A.F. (1987), Parsimonious Modeling of Yield Curve, *Journal of Business,* 60, 473–489.

Schaefer, S. M. (1981). Measuring a tax-specific term structure of interest rates in the market for British Government securities, *Economic Journal*, 91, 415–38.

Stander Y. (2005), *Yield Curve Modeling.* Palgrave Macmillan.

Svensson L.E. (1994), Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994., Centre for Economic Policy Research, Discussion Paper 1051.

Svensson L.E. and Söderlin P. (1997), New Techniques to Extract Market Expectations from Financial Instruments, Centre for Economic Policy Research, Discussion Paper 1556.

Svensson L.E.O. (1996), Estimating the Term Structure of Interest Rates for Monetary Policy Analysis, *Scandinavian Journal of Economics*, 98, 163-183.

Vasicek O. (1977). An equilibrium characterization of the term structure, *Journal of Financial Economics* 5, 177–188.

Vasicek O.A. and Fong H.G. (1982), Term Structure Modeling Using Exponential Splines, *Journal of Finance,* 37, 339–348.

Wahlstrøm, R. R., Paraschiv, F., and Schurle, M. (2021). A Comparative Analysis of Parsimonious Yield Curve Models with Focus on the Nelson-Siegel, Svensson and Bliss Versions. Computational Economics, March 2021.

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